

A DECISION MODEL FOR INVESTMENT ALTERNATIVES
IN HIGHWAY SYSTEMS

by

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ABSTRACT

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Submitted to the Department of Civil Engineering on August 14, 1972 in partial fulfillment of the requirements for the degree of Doctor of Science.

The performance of a large class of public facilities is dependent upon the subjective evaluation of the users and their relative acceptability of these facilities. Thus, the specific goals of these systems are derived from the more general goals of the society comprising the users of the systems. In this context, this study presents a framework for the evaluation and optimal selection of highway pavements stemming from the levels of services they provide to the users during their operational life.

The serviceability level of any structural system in an operational environment is bound up by uncertainties resulting from the randomness in both the physical characteristics of the system, and the surrounding environment. These uncertainties are expressed in terms of the system's reliability, i.e., the probability that the system is providing satisfactory levels of serviceability throughout its design life. The levels of maintenance exercised on the system control its serviceability level as well as its reliability and the extent of the operational lifetime. Further, economic constraints are important factors which determine the levels of serviceability and reliability of the system by controlling the initial construction costs as well as maintenance and vehicle operating costs.

The pavement design model presented in this study accounts for the interactions which exist among the materials, environmental, and economic attributes of the system. In this respect, design is viewed as a process of sequential evolution of systematic analyses whose ultimate goal is the achievement of an optimal design configuration. This represents a departure from the conventional design methods for these systems.

The design process is realized through the implementation of three phases of analysis. One is concerned with the selection of materials, and the evaluation of the structural behavior of the system in a simulated operational environment. The second phase deals with the evaluation of serviceability, reliability, and maintenance levels throughout the life of the system. Finally, the third phase addresses itself to the management issues of the system, in terms of the choice of optimal maintenance policies, and decisions related to cost optimization and alternative design tradeoffs.

A simple illustration and a limited sensitivity study are presented to demonstrate the capability of this model to predict the serviceability, reliability and life of the system. Further, a numerical example for the selection of alternative maintenance policies is presented.

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I. INTRODUCTION

Highway systems belong to a large class of public facilities whose specific goals and functions are derived from the more general goals of the society comprising the users of the systems. The performance of these systems is largely dependent on the subjective evaluation of the users and their relative acceptability of the systems. It is therefore desirable to evaluate these systems from the standpoint of the levels of services they provide at any time during their operational life. In this context, failure may be viewed as a threshold that is reached as the performance level exceeds some unacceptable limits viewed by the users of the facility.

The present design practices for highway systems are largely empirical, based on experience and engineering judgement. They are basically expressed in terms of correlations between soil type, base course properties, thickness of the different layers, and traffic characteristics. Although these methods have met in the past with moderate success, the rapid changes in traffic volume, in construction and maintenance costs and techniques, and the use of potentially new materials make experience and empiricism obsolescent or totally lacking. Therefore, a design method which combines theory and empiricism to a lesser extent is needed. The method must encompass a set of analytical procedures that can effectively simulate the

behavior of the system and the interactions among its components. Further, this design method must choose as a means of system evaluation such measures of effectiveness that define the specific goals and functions of the system it represents.

I.1. Measures of Effectiveness

The analysis and design of pavements, similar to the analysis and selection of investment opportunity, require a knowledge of both the supply and demand functions involved. In this context, the supply functions may be considered as a set of techniques available to combine a variety of resources to produce highway pavements. A set of resources combined in a particular way is referred to as a strategy. For example, a particular combination of certain types of material in a given geometrical configuration constitute one strategy. On the other hand, placing different types of material in another geometrical configuration forms another strategy. Usually, there are several strategies that can be acceptable for any situation. The question would be which strategy meets the demand requirements most efficiently, where efficiency can be normally translated in economic terms.

The demand function for pavements can be expressed in terms of the three components of performance; serviceability, reliability, and maintainability (S-R-M).

The level of performance or serviceability of any system, functioning in an optimal environment, is bound up by the uncertainties inherent in the physical characteristics of the system and in the surrounding environment. These uncertainties can be expressed in terms of the reliability of the system, i.e., the probability of providing satisfactory levels of performance at any point within the operational life of the system.

Maintenance efforts exercised throughout the lifetime of the system controls the level of performance of the system and its reliability, as well as its operational life. This may be expressed in terms of the maintainability, which is a measure of the effort required to maintain adequate levels of serviceability throughout the design life of the system (L2)*.

Economic constraints play an important role in controlling the levels of serviceability throughout the lifetime of the system by determining the initial costs, maintenance costs, and vehicle operating costs.

I.2. Goal Formulation

The levels of (S-R-M) for each pavement should be commensurate with the type of highway involved. For example when the pavement is for an expressway, its levels of (S-R-M) are quite different than those for a rural or suburban road.

* The numbers in brackets refer to the list of references

For an expressway in a metropolitan area one demands a very slow rate of drop in its serviceability, a very high degree of reliability that something serious may not go wrong with the pavement, and a very low maintenance so that the traffic will not be disrupted. However in rural areas one may tolerate lower reliability and more dependence upon maintenance for keeping the pavement operational. These variations in demand for the two cases are obviously dictated by the economics of the two roads. The monetary loss and social consequences of closing an expressway are more intolerable and expensive than those of rural road.

The design decision is then to choose the strategy which meets the demand requirements subject to certain constraints. These constraints can be economic or otherwise. For example the constraint may be to choose the alternative which costs the least, or to choose one which needs minimum maintenance.

In order to be able to predict that a certain pavement system (a strategy) will meet the demand requirements, it is essential to have analytic or empirical means to assess how the given pavement system will perform in the specified environment and the projected loading conditions. Most empirical means (which, in the literature are erroneously referred to as methods of design) attempt to assess the performance capability of a given pavement system by simply evaluating a single response of the system. For example,

they indicate that a given pavement system will perform satisfactorily if a maximum allowable stress or deformation at certain points of the pavement is not exceeded. These maxima or limits are often set based on field observations and past experience. Their application to other locality and their usefulness under a different set of circumstances have always been questionable.

Assuming that the "society" makes optimal use of its resources in deriving the maximum overall benefits for any of its commodities, the overall objective in the design of highway system may be stated as: providing an economical riding surface at an "adequate" level of performance and reliability for an optimum time period. Such adequate levels result in a structure which is reliable, safe in terms of comfort and frictional characteristics, and which will maintain some structural integrity at a low cost to the "society" (M3, M4). The above statements define a goal space from which a designer can derive his design objectives and requirements. In this capacity, the pavement designer is concerned with the description and selection of a set of optimal actions fulfilling the requirements imposed by the goal space and constrained by the allocated resources. Moavenzadeh and Lemer (M3) provide a methodology for defining a goal space for pavement design in the form of a hierarchical structure. This structure can then be decomposed into less complex elements as a basis for search of alternate

goals and specific solutions (M4).

I.3. The Proposed Design Framework

This study presents a methodological framework for the analysis and selection of pavement systems suitable for a given set of goals and constraints. A set of models and algorithms has been developed at two different levels of analysis: analysis of the physical behavior of the system, and analysis for the selection and optimization of a design system. The first involves a set of mechanical and phenomenological models which describe the response of the system in a realistic operating conditions of traffic and environment. From these models the progression of damage within the system can be evaluated using some physical transfer functions. The second level of analysis utilizes the above information to determine the level of services that the system is providing at any time and the reliability of the system in the operational environment. Maintenance policies can be generated, and evaluated, and an optimum set of strategies may be selected for a given design configuration over the lifetime of the system. Similarly, alternative design configurations can be generated and evaluated, and a framework for the selection of optimum systems is presented based on cost criteria and users' constraints.

The basic features which characterize this study can be summarized as follows:

1. One particular feature of this study is that the proposed method of design for structural systems represents a departure from the traditional cook-book style methods generally pursued in the literature. Instead, the design is viewed as a process of sequential evolution of systematic analyses whose ultimate goal is the achievement of an optimal design configuration.
2. The criteria for model selection and evaluation are based on the users' subjective preferences for constructed facilities derived from their particular needs and sets of values. From this standpoint, the highway pavement is viewed as a system which is providing certain services to its users, and the quality of providing these services must be evaluated from the users' demands and preferences
3. The models cover a wide spectrum of activities encompassing a large body of knowledge ranging from rational and applied mechanics to probability and operations research disciplines. The particular advantage derived from this coverage is that it provides a means of continuity and integrity to the analysis and design. One can study for example the influence of change in geometry, physical properties, traffic patterns or quality control levels not only on the future behavior of the system, but also on its life, maintenance policy, costs, and so forth, through a relatively simple process.

4. The models possesses a causal structuring thereby defining the different interactions between the system and the surrounding environment. Also, the feedback processes resulting from maintenance activities are accounted for.
5. The models recognize and incorporate the elements of uncertainty associated with the natural physical phenomena and processes represented.

The following chapter deals with the development of a set of models which are concerned with the prediction of the behavior of the system in the operational environment. A structural model, based on mechanistic theories is used for the analysis of the structural response of the system expressed in some physical manifestations of damage. These damage manifestations are similar to those developed by AASHO [A1] as the components of damage that the users generally are sensitive to. The system itself is represented by a three-layer viscoelastic model with each layer having certain statistical properties and geometry. A serviceability maintenance model utilizes the information provided by the structural model to predict stochastically the serviceability, reliability and life of the system allowing for maintenance activities at desired time periods to be exercised to upgrade the system.

A framework for a decision structure for the choice of optimum maintenance strategies for a certain design configura-

tion is presented in Chapter III. A dynamic programming-based algorithm is developed for this optimal selection.

Further a limited sensitivity analysis is presented in Chapter IV to validate and demonstrate the effectiveness of the developed models, which have been coded into a set of computer programs.

The development of further research activities to complement and calibrate these models in order that they may be effectively used as practical design tools is discussed in Chapter VI.

II. AN ANALYTIC MODEL FOR PAVEMENT PERFORMANCE

II.1 An Overview

The development of a rational method for the analysis and design of engineering systems involves a set of procedures, of which the selection and analysis of a model or prototype for realistic inputs constitute a major part. A model, in this context, is an abstract representation of the form, operation, and function of the real or physical system (D1). To provide such a model for design purposes, it is essential that the system be characterized realistically; information about the design functions, behavioral interactions, and failure patterns and mechanisms must be provided.

The analysis of a systems-oriented problem requires the applications of certain procedures, each of which involves the use of different methods and techniques for problem-solving and formulation. Therefore, the main problem at the outset of the analysis is that of "proper" modelling. Different analysts may formulate different models for the same system. It is not very clear whether the problem is that of formalism or that of abstraction. It is also possible that subjectivity and arbitrariness in definitions of the different aspects of formal problems may result in non-unique solutions to such problems. In any case, what follows is that subsequent decisions on the configuration of a certain design problem may vary considerably among analysts depending on the different interpretations and valuation of the existing information. Since this is an unavoidable problem, one must at least make sure that such interpre-

tations are based on a more objective source of information which may serve as a guide for one's actions and decisions.

One of the major objectives in the area of large-scale systems modelling is the construction of a causal model capable of handling the interaction of the different components of the system it represents. A causal model is one which is based on an a priori hypothesis of the system's behavior as well as the interaction, within the system, of the different excitations and system's characteristics. Such an interaction occurs in accordance with certain functional relationships which define the behavior of the system and its responses in terms of some physical transfer functions.

In general, most systems analysts require the following systematic procedures for the solution of systems-oriented problems:

1. Problem definition: which involves the determination of the overall systems requirements, objectives, and constraints. Factors such as performance, reliability, cost, maintainability, and life expectancy are taken into account as measures of effectiveness for the evaluation of the system.
2. Generation of Alternatives: where several solutions are synthesized to form a solution space, which is scanned for the choice of a feasible solution.
3. Synthesis of the system: which involves the complete theoretical and physical design of the system.

4. Evaluation of alternatives: where the information about the characteristics of the synthesized system is updated, possibly through simulation. In general, as the continuous process of evolution of the system proceeds, more and more emphasis is shifted from the theoretical representation of the system and placed on its physical aspects.
5. Testing of the system: where the characteristics of the resulting system are determined for the overall evaluation of the system.
6. Refinement of the design: in which the systems requirements are correlated to the test data obtained above to re-examine the overall system interrelationship and reassess the contribution of its individual components. In this context, it may result that certain goals are not achievable and require further examination and change (S1).

This chapter discusses the development of a set of models for the analysis of highway pavement within its operational environment. The models are intended to be used in conjunction with the design of pavement systems. In this context, the design process is viewed as a process of evolution of the analysis in which alternatives are synthesized and evaluated for the choice of optimal design. The operational policies encountered in the handling and maintenance of the system are a part of this design methodology. The choice of feasible and optimal selections is based on a set of criteria which are partly subjective in nature and incorporate the users' demands and

aspirations for the systems at hand.

II.2 Framework of the Model

The general framework of the model may be described as a management-oriented framework. The overall model is viewed as having three subsets of models which, when integrated together, result in a predictive structure for pavement design. These subsets are categorized into: structural model, serviceability-maintenance model, and cost model.

These models and an optimization sub-model would then provide a basis for the selection of an optimal system, among alternative configurations and maintenance policies, based on some cost-effectiveness analysis.

The overall model developed in this study is shown in a block structure in figure (1). This thesis has been concerned with the development of the first two models which are discussed below.

II.2.1 Structural Model

The structural model is concerned with the analysis of the structural response of the pavement systems. It provides information about the response and damage of the system with time. For analytical convenience, it has been decomposed into two stages: primary response and ultimate response.

A three-layer viscoelastic model is used to represent the pavement structure. This model is subjected to loading patterns similar to those of typical traffic patterns. The effect of climatic

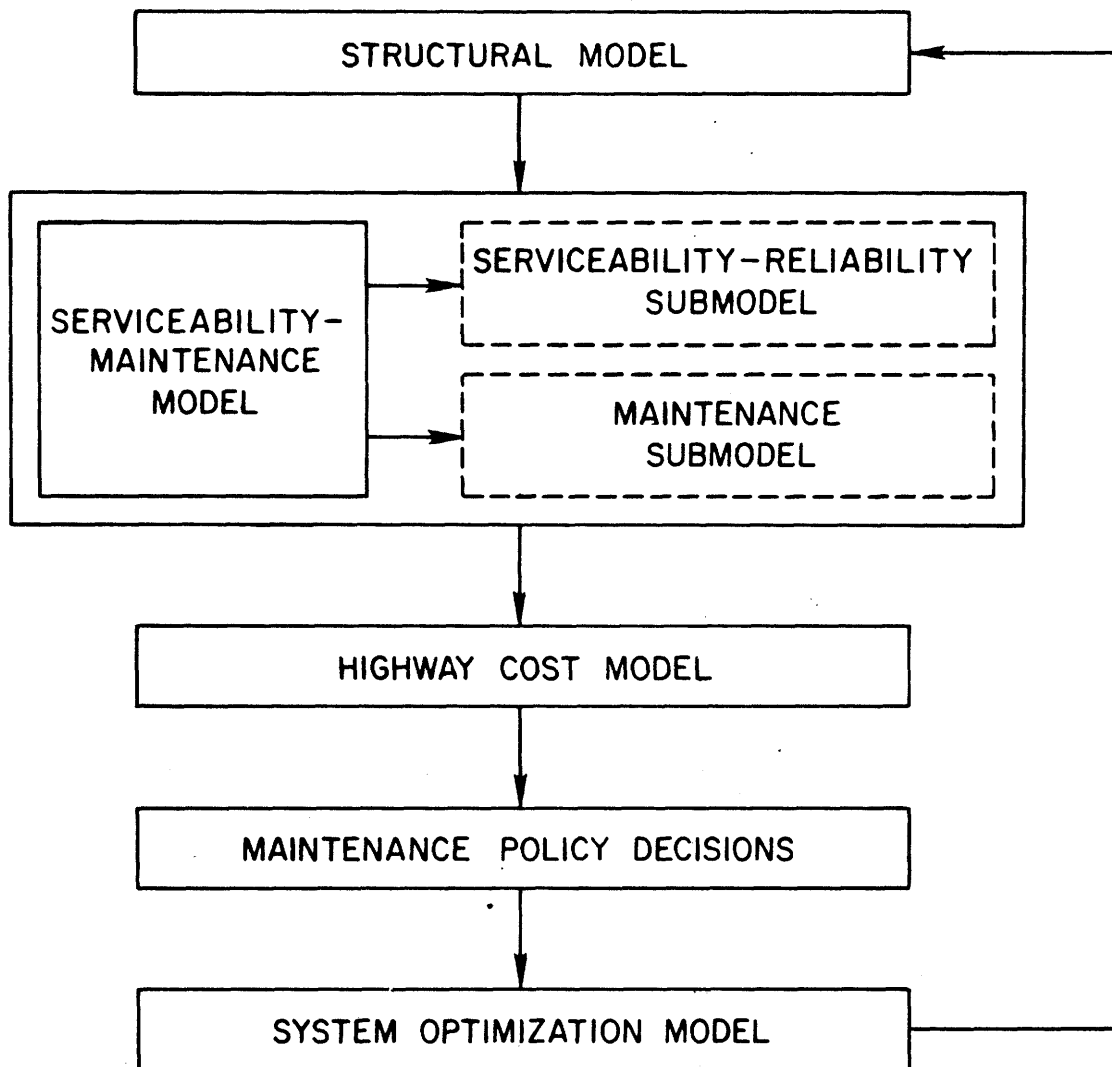


FIGURE 1. AN OVERALL DESIGN MODEL.

variations in the form of temperature templates representing regional conditions are also accounted for.

In the primary response stage, statistical information regarding the response of the system to loads and environment is obtained. The response of the system is expressed in terms of first and second order moments of histories of stresses, strains, and deformations at any point within the layered system. (A2, E1, F1)

In the ultimate response stage, the primary responses of the system are expressed in terms of some descriptive, less objective damage indicators utilizing some probabilistic transfer functions. Such indicators reflect the user's perception and sensitivity to the quality of the system, similar to those developed by the AASHO Road Test (A1). These indicators define the surface quality of the system and are expressed statistically in terms of cracking and longitudinal and transverse deformation.

The structural model is depicted in Figure (2).

II.2.2 Serviceability-Maintenance Model

This model is in turn decomposed into two submodels: a serviceability-reliability (S-R) model, and a maintenance model.

In the S-R model, the distress indicators obtained from the structural model are combined in a regression form to provide a set of numerical values indicating the levels of serviceability of the system and the user's relative acceptability of these levels. One form of such an equation has been developed by AASHO which refers to the level of services provided by the system to the users

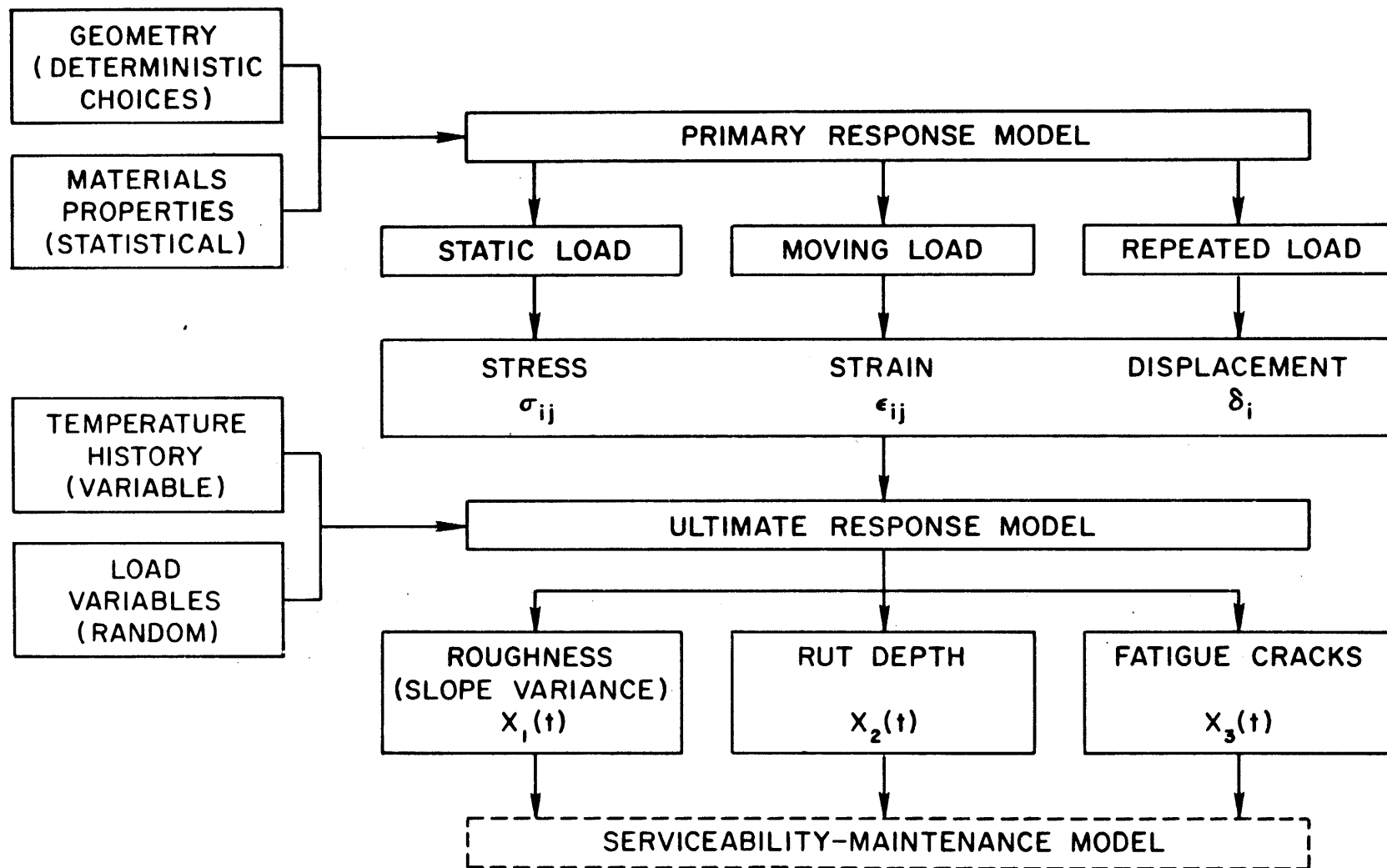


FIGURE 2. FLOW CHART OF THE STRUCTURAL MODEL.

as the Present Serviceability Index (PSI). This equation has been reformulated in light of the uncertainty associated with the damage characteristics, and probabilistic estimate of the serviceability index are obtained at any desired time. From this, one can determine the probabilities of having any value of the PSI, referred to as the state probabilities. The probability, at any time, of being above some unacceptable value of the PSI is defined as the reliability of the system. This is a measure of the level of confidence that the system is performing its stipulated design functions as viewed by its users. In this context, the life expectancy of the system is determined by the model based on its serviceability and reliability.

The maintenance model is aimed at introducing activities which result in improvements of the level of serviceability of the system, and at studying of the influence of such activities on the future behavior of the system and its life expectancy. In this model, strategies are generated over a range of time spectra, and their subsequent effects on the serviceability, reliability, and life are determined at some associated cost estimates.

The serviceability-maintenance is shown in the flow diagram of Figure (3).

II.2.3 Cost Model

The cost model, which has been developed elsewhere (A3, M1), addresses itself to the determination of the total costs of construction, operation, and maintenance of highway systems. It incorporates three components: construction costs, roadway maintenance costs, and

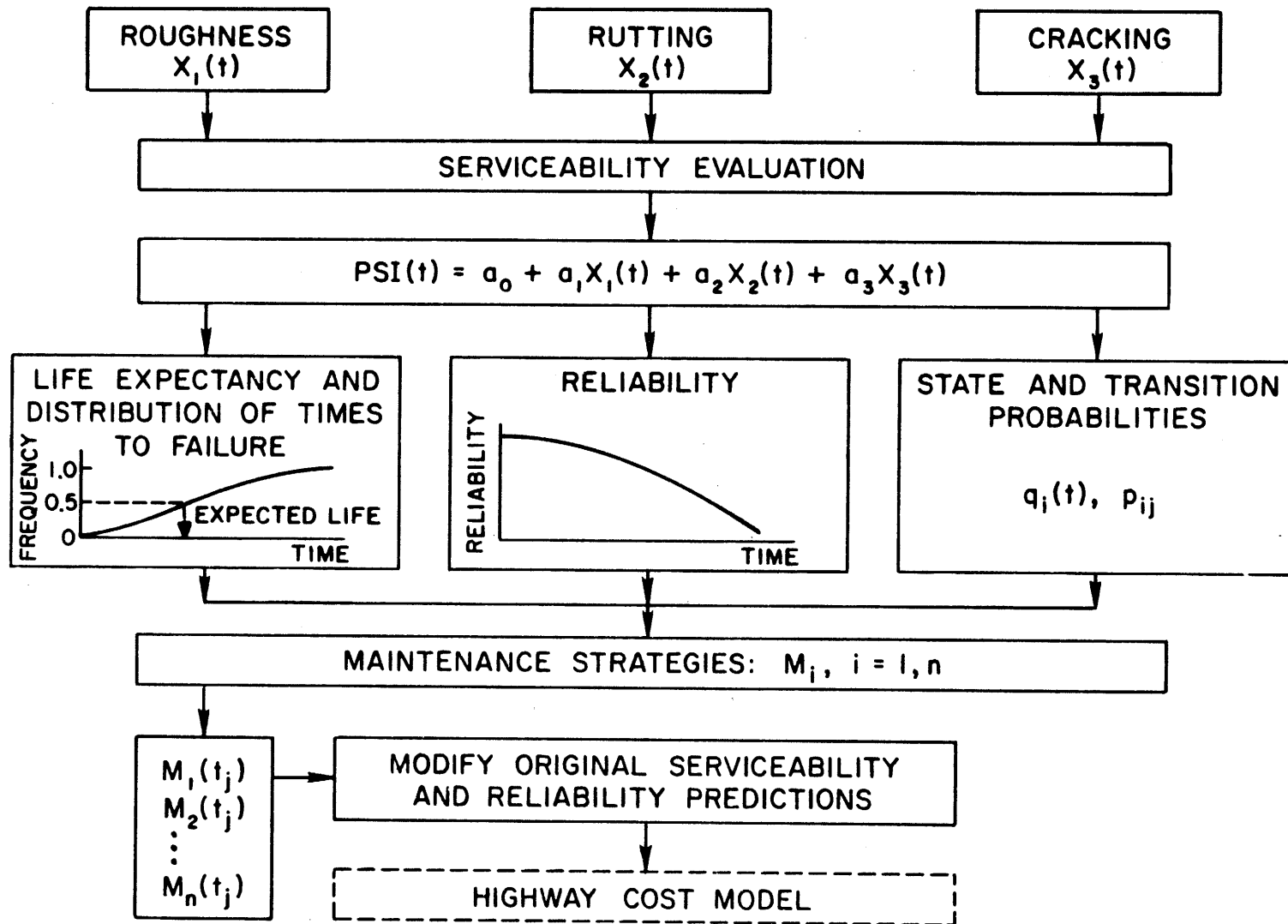


FIGURE 3. SERVICEABILITY-MAINTENANCE MODEL.

vehicle operating costs. Each component in this model provides estimates of the resource consumption, and yields the money costs of these resources using separately defined prices. Therefore, this model is adaptable to any economy regardless of the relative costs of different resources.

This model may be used in conjunction with the above models to provide a basis for decisions on the selection, operation, and evaluation of optimal design systems. The flow diagram in figure (4) depicts the nature of this model.

In the following sections, these models are discussed in details within the above framework, and the mathematical formulation and the related assumptions are presented. The cost model is not a part of this study, and therefore was not implemented as a part of the present analysis. However, the potentiality of integrating it with the other models has been examined and few modifications may be necessary to achieve an overall design system as was discussed earlier.

II.3 Structural Model-Nature and Operation

The structural model is a mathematical model of the pavement structure. It consists of a three-layer elastic or viscoelastic system utilizing mechanistic theories for prediction of pavement response and distress. For analytical convenience, this model is divided into two stages: primary and ultimate. The primary stage has been developed by Ashton (A2) and Elliott (E1) to yield stresses, strains, and deformation at any point within the layered system with time under deterministic operational conditions. These conditions being (1) a static load applied in a step-form and kept for an

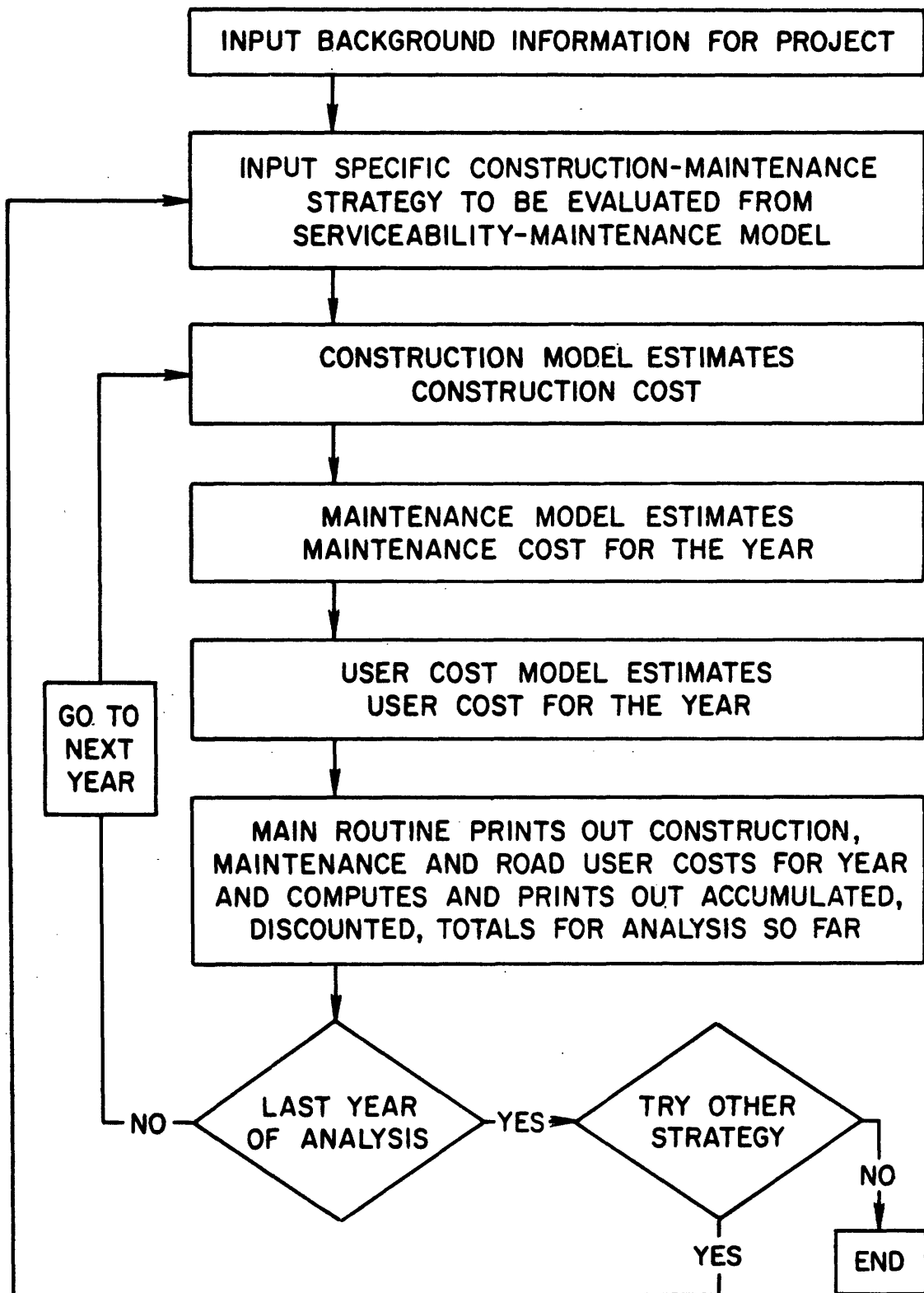


FIGURE 4. OPERATION OF THE TOTAL COST MODEL.
(REFERENCE A3)

indefinite period of time, (2) a constant load moving along a straight line at a surface at a fixed velocity, and (3) a load repeatedly applied in the form of a haversine wave with fixed duration, amplitude, and frequency.

This earlier work has been modified to account for the realistic random nature of the operational environment. It is recognized that the behavior of pavement systems in an operational environment is largely a function of the intrinsic characteristics of the system, as well as the load and environment to which the system is subjected. Traffic loading is far from being predictable at any instance in time both in magnitude and frequency. Also, the climatic environment surrounding the system changes in a somewhat random manner that can only be statistically defined and characterized.

Similarly, the properties of the materials in the different layers vary considerably from one point to another due to variations in mixing and fabrication practices which introduce some inhomogeneities in the materials.

These uncertainties result in an unpredictable behavior of the system associated with probabilities of overload or inadequate capacity of the system to carry on its stipulated functions. With this frame of mind, a simulation approach was developed to account for these uncertainties under static loading modes (F1). The inputs and outputs are described in this approach in terms of probabilistic distributions instead of single-valued estimates.

The present study utilizes the information obtained from the above simulation analysis to provide probabilistic estimates of

the history of response characteristics under randomly repeated loads and varying temperatures. The general probabilistic solutions mentioned above and the associated assumptions are detailed in the following sections.

II.3.1 Probabilistic Analysis of the Structural Model

The following analysis is conducted for the case where loads are applied randomly on the system in a manner commensurate with actual vehicular arrival patterns in the real system. It utilizes the output of the simulation for the static load case, where a set of response functions are obtained in statistical formats, i.e., in terms of first and second order moments of the step function response. This response may be viewed as a characteristic function for the total pavement system in which the contribution of all components is integrated as a single response to a step function. This response can be expressed in the form of an exponential series:

$$\sum_i \eta_i \bar{G}_i \exp(-t\delta_i) \quad (2.1)$$

where η_i is a random with a mean of 1.0 and a variance equal to the square coefficient of variation of G_i ,*

\bar{G}_i is the mean value of the random variable G_i , and δ_i are the exponents of the series**, taken as deterministic quantities.

$$\text{*Coefficient of Variation of } G_i = V_{G_i} = \frac{\sigma_{G_i}}{\bar{G}_i}$$

** δ_i 's are actually correlated to the retardation times of viscoelastic systems.

To simplify the analysis considerably, an assumption has been made in regard to η_i ; that $\eta_i = \eta$, for all i . This assumes that the response functions of the pavements to a step load are well behaved and that they do not criss-cross each other due to variations in the materials properties.

The terms in equation (2-1) are used as inputs to the next stage of the analysis, i.e., the repeated random loading mode. This state, as has been mentioned earlier, takes into consideration the fact that vehicular loads are applied randomly on the pavements with varying velocities and intensities. It also utilizes the time-temperature superposition principle to account for the variations in the response due to change in temperature histories throughout the service life of the system. The effects of moisture have not been incorporated in the present study, but can be implemented in a manner similar to the temperature case.

II.3.2 Assumptions and Propositions:

Before turning into the formalization of the model, the assumptions that led to this analysis are discussed below. These assumptions may be divided into two categories: those related to the input variables, and those related to the output variables

A) Input Variables:

1. Traffic Load: The application of traffic load on a pavement system has been assumed as a process of independent random arrivals. Vehicles arrive at some point on the pavement in a random manner both in space (i.e., amplitude and velocity), and in time (of arrival).

The arrival process is modelled as a Poisson process with a mean rate of arrival λ . The probability of having any number of arrivals n at time t may be defined as:

$$P_n(t) = \frac{\exp.(-\lambda t) (\lambda t)^n}{n!} \quad (2-2)$$

Assumptions of stationarity, nonmultiplicity, and independence must be satisfied for the underlying physical mechanism generating the arrivals to be characterized as a Poisson process (B1). In this context, stationarity implies that the probability of a vehicle arrival in a short interval of time t to $t + \Delta t$ is approximately $\lambda \Delta t$, for any t in the ensemble.

Nonmultiplicity implies that the probability of two or more vehicle arrivals in a short interval is negligible compared to $\lambda \Delta t$.

Physical limitations of vehicle length passing in one lane on highway support this assumption; it is not possible that two cars will pass the same point in a lane at the same time.

Finally, independence requires that the number of arrivals in any interval of time be independent of the number in any other nonoverlapping interval of time.

In a Poisson process, the time between arrivals is exponentially distributed. This property is used to generate a random number of arrivals within any time interval t .

The amplitudes of the loads in this process are also statistically distributed in space. Traffic studies (D3) have shown that a

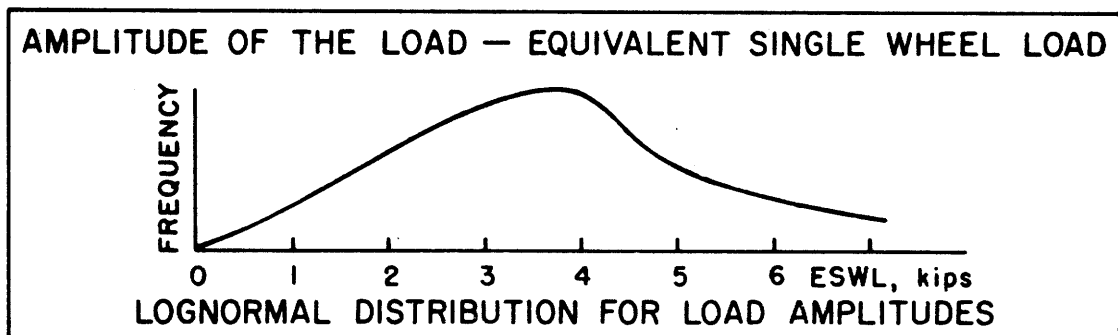
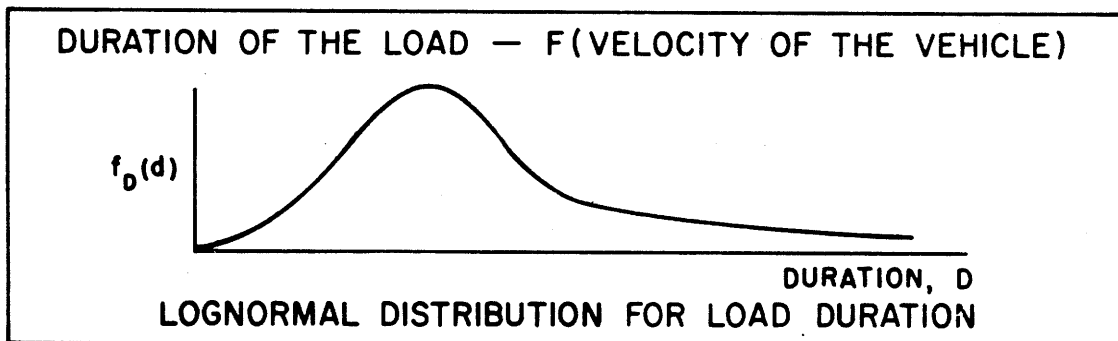
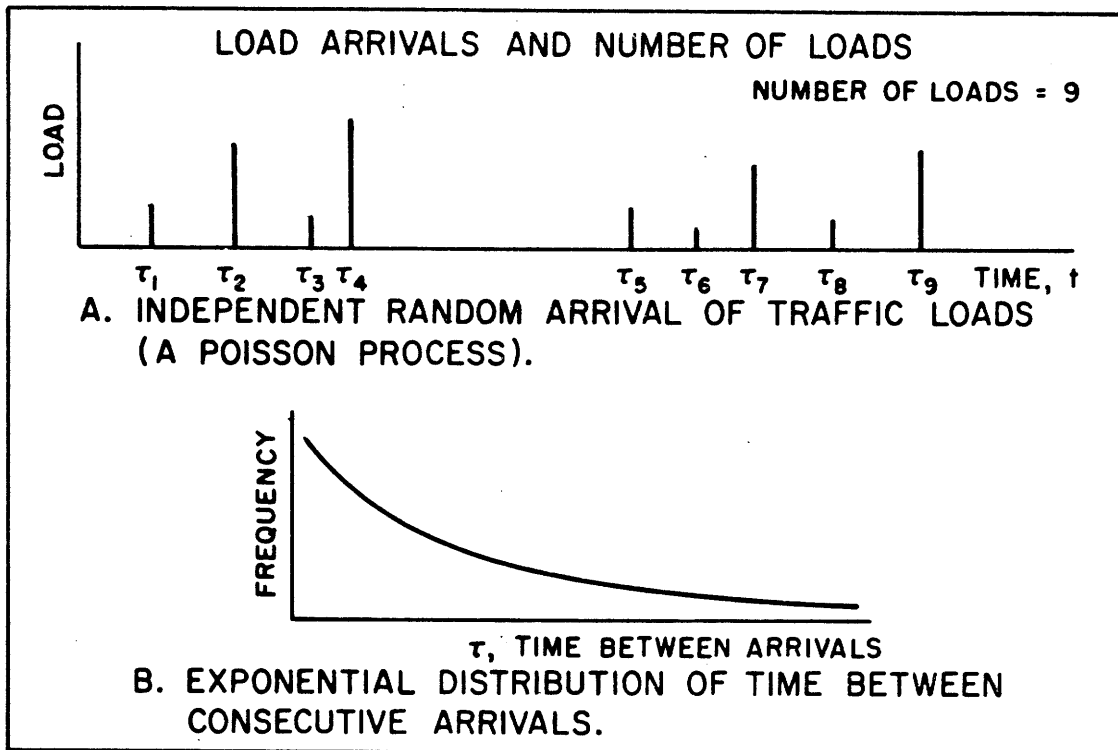


FIGURE 5. DISTRIBUTION OF LOAD CHARACTERISTICS.

logarithmic-normal (lognormal) distribution is suitable to represent the scatter in load magnitudes. Means and variances of load amplitudes are used to represent this scatter.

The load duration, a function of its velocity on the highway, is also a random variable. In a typical highway for example, speeds may vary from 40 to 70 miles per hour. Accordingly, the load duration was assumed to have a statistical scatter represented by its means and variances from distributions obtained by traffic studies.

Figure (5) shows the statistical characteristics of typical load inputs to the model.

2. Climatic Environment: In this attribute, only temperature effects have been considered, with the assumption that moisture can be incorporated in a similar fashion at a later stage. Temperature variations from one period to another are accounted for through the time-temperature superposition of the response of the system. The variations within these periods have not been considered due to the complexities they introduce to the analysis. One can choose the time periods in such a way that averaging over temperatures within these periods can be justifiable. The present study allows for the study of hourly, daily, weekly, monthly, quarterly, and yearly intervals of time.

3. System Characterization Function: This set of inputs describe the characteristic response of the total system to a step function, and is statistically described by a set of random coefficients (G_i) and exponents (δ_i) of an exponential series of the form described by

equation (2.1) above. This is obtained by simulation of static load response, as has been discussed earlier in this section.

A typical response equation to a random load history and temperature history can be expressed by a convolution integral of the following form:

$$R[t-\tau, \phi(\tau)] = \alpha[\phi(t)] + \beta[\phi(t)] + \int_0^t \left[\sum_{i=1}^N G_i \exp(-t^* \delta_i) \right] d\tau - \int_{-\infty}^t \left[\sum_{i=1}^N G_i \exp(-S^* \tau_i) \right] d\beta[\phi(S)] \quad (2-3)$$

The second integral in this equation is only a corrective term and can be neglected for all practical purposes.

where:

$$t^* = \int_{\tau}^t \gamma(x) dx$$

α , β , and γ are mapping parameters

for temperature effects in which:

α = factor for vertical change of scale = 0 in this study.

β = vertical shift factor = $T(t)/T_0$

γ = horizontal shift factor

= $10^{.162 (T - T_0)}$

T_0 = reference temperature in °K

$\phi(\tau)$ is a vector representing the temperature history

and $R[,]$: represents a vector for the system's response history

This response is expressed (through probabilistic transfer functions, which will be described below) in terms of the damage indicators suggested in the AASHO serviceability evaluation (A1). The propositions and assumptions made in this study for the evaluation of these damage components are listed below..

b) Output Variables

The output variables are expressed in terms of two damage manifestations in the pavement structure: cracking and deformation. Deformation develops in the pavement in the transverse and longitudinal profiles of the pavement. One is manifested by the rutting in the wheel paths of the vehicles, and the other in the roughness of the pavement longitudinal profile, measured by the slope variance of the profile. The mechanisms of development of each of these damage manifestations are described below.

1. Rutting: This component is assumed to be primarily the result of a channelized system of traffic thereby causing differential surface deformation under the areas of intensive load applications in the wheel-paths. Given the statistical characteristics of the road materials and of the traffic, one can determine this component, measured by the rut-depth, from the spatial properties of traffic loads. For a given traffic pattern, the split Poisson property is invoked*, to obtain the differential surface deformation due to the channelization of the traffic.

*This property states that if a stochastic process is of the Poisson type with a mean of λ , then the arrival pattern will still be of the Poisson type if there is a split or addition to the event sequence with a modified mean λ' .

2. Roughness: This component defines the deformation along the longitudinal profile of the pavement. To obtain some measures of roughness, information about the spatial correlation of the properties of the system must be obtained. This can be expressed in terms of the autocorrelation function of the surface deformation. This implies the assumption that roughness in pavement is mainly caused by the variations in the properties of the materials and fabrication methods. One can relate the spatial variations in the materials to those in the surface deformation along the pavement profile. In this study, the slope variance as used by AASHO is used as a measure of roughness of the pavement, as will be described later in this chapter.

3. Cracking: Cracking is a phenomenon associated with the brittle behavior of materials. A fatigue mechanism is believed to cause progression of cracks in pavements. In this study, a phenomenological approach has been adopted, namely a modified stochastic Miner's law for progression of damage within materials. This has been used in conjunction with a healing mechanism for viscoelastic materials at suitably high temperatures. It is recognized, however, that a probabilistic microstructural approach based on fracture mechanics can provide a better substitute for the prediction of crack initiation and progression within the pavement structure.

Having reviewed the assumptions upon which the analysis in this study has been established, the basic formulations and analyses are presented in the following subsection.

II.3.3 Probabilistic Analysis - General Response Formulation

The following convolution integral can be used to represent the response of the system to load and environment:

$$R[t-\tau \underset{t=-\infty}{\overset{t}{\phi}}(\tau)] = \int_{-\infty}^t f(t, \tau, \phi) \frac{\partial P(\tau)}{\partial \tau} d\tau \quad (2-4)$$

where $f(\cdot)$ represents the response function, and $P(\cdot)$ is the loading function.

For a haversine ($\sin^2 \omega \tau$) load, one can expand equation (2.4), with reference to equation (2.3) above as follows:

$$R(t) = \alpha [\phi(t)] + \beta[\phi(t)] \int_0^t \sum_{i=1}^N -G_i \exp(-t*\delta_i) A(\tau) * \left\{ \frac{\sinh \left[\frac{\gamma(\tau) \delta_i D(\tau)}{2} \right]}{1 + \left[\frac{\gamma(\tau) \delta_i D(\tau)}{2\pi} \right]^2} \right\} d\tau \quad (2-5)$$

Equation (2.5) may be broken into a sum of integrals over (L) periods of time, each of which having a constant (average) temperature:

$$R(t) = \sum_{k=1}^L \beta[\phi(f_L)] \int_{t_{k-1}}^{t_k} \sum_{i=1}^N -G_i A(\tau) \frac{\sinh \left\{ \frac{\gamma_k \delta_i D(\tau)}{2} \right\}}{1 + \left\{ \frac{\gamma_k \delta_i D(\tau)}{2\pi} \right\}^2} * \exp \left[-\delta_i \left\{ (t_k - \tau) + \sum_{p=k+1}^L \gamma_p (t_k - t_{k-1}) \right\} \right] d\tau \quad (2-6)$$

Assuming that $\alpha[\phi(t)] = 0$ (2-7)

and letting $t_k - t_{k-1} = t_{DEL}$, for all $k = 1, 2, 3 \dots$, (2-8)

and since traffic loads arrive in a Poisson process integrally, therefore equation (2.6) becomes:

$$R(t) = -\sum_{k=1}^L \sum_{j=1}^{\sum_{i=1}^L} \beta[\phi(t)] G_i A_{jk} \frac{\sinh \frac{\gamma_k \delta_i D_{jk}}{2}}{1 + \left[\frac{\gamma_k \delta_i D_{jk}}{2\pi} \right]^2} \quad (2-9)$$

$$\cdot \exp \left[-\sum_{p=k}^L \gamma_p t_{DEL} \delta_i \right] \cdot \exp(\tau_j \gamma_k \delta_i)$$

where:

A_{jk} = Load Amplitude

D_{jk} = Load Duration

n_k = total number of Poisson arrivals in the k^{th} period

$\beta(\)$ and γ_k are temperature shift factors as defined in the previous section.

For simplicity of notation in further equations, let us call

$$\beta_L = \beta[\phi(t_L)] \quad (2-10)$$

The expected value and variance of the response of the system to a load excitation and temperature history of this derivation may be found in Appendix I. This derivation follows from equation (2.8) which is a sum of "compound filtered Poisson processes". This is of the general form:

$$R_k(t) = \sum_{-\infty < \tau < \infty} \omega(t, \tau, \phi) \quad (2-11)$$

For which, Parzen [P1] presents a general solution in terms of first- and second-order moments and correlations as follows:

$$E[R_k(t)] = \lambda \int_{-\infty}^{\infty} E[\omega_k(t, \tau, \phi)] d\tau \quad (2-12)$$

$$\text{Var}[R_k(t)] = \lambda \int_{-\infty}^{\infty} E[|\omega_k(t, \tau, \phi)|^2] d\tau \quad (2-13)$$

$$\text{Cov}[R_k(t), R_m(s)] = \lambda \int_{-\infty}^{\infty} E[\omega_m(s, \tau, \phi) \omega_k(t, \tau, \phi)] d\tau \quad (2-14)$$

where, λ is the mean rate of the Poisson arrivals.

The definitions for the terms in equation (2.12) through (2.14) and what follows may be found in Appendix I. From the above equations, as explained in the appendix, one can write

$$E[R(t)] = -\beta_L E\left[\sum_{k=1}^L R_k(t)\right] \quad (2-15)$$

$$E[R(t)] = -\beta_L \sum_{k=1}^L E[R_k(t)] \quad (2-16)$$

and

$$\text{Var}[R(t)] = \beta_L^2 \text{Var}\left[\sum_{k=1}^L R_k(t)\right] \quad (2-17)$$

$$\begin{aligned} \text{Var}[R(t)] = & \beta_L^2 \left\{ \sum_{k=1}^L \text{Var} [R_k(t)] \right. \\ & \left. + 2 \sum_{k=1}^L \sum_{m=k+1}^L \text{Cov}[R_k(t), R_m(t)] \right\} \end{aligned} \quad (2-18)$$

The analysis in Appendix I, yields the following solutions for (2-16) and (2-18)

$$E[R(t)] = -\beta_L \sum_{k=1}^L \lambda \bar{A} \sum_{i=1}^N \bar{G}_i V_{ik} \bar{\phi}_{ik} \omega_{ik} \quad (2-19)$$

$$\begin{aligned} \text{Var} [R(t)] = & \beta_L^2 \sum_{k=1}^L \lambda [1 + \sigma_\eta^2] \{ (\sigma_A^2 + \bar{A}^2) (I_k' + I_k^2) \\ & + \lambda \bar{A}^2 I_k^3 \} + 2 \sum_{k=1}^L \sum_{m=k+1}^L \lambda (\bar{A}^{-2} + \sigma_\eta^2) \\ & \sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j V_{ik} V_{jm} \frac{1}{2} \\ & \{ [J_{km}^2 - J_{km}^4] \phi_{km} + 5(\bar{D}) \sigma_D^2 \} \end{aligned} \quad (2-20)$$

where the second term in equation (2-20) is the

$$\text{Cov} [R_k(t), R_m(t)]$$

All the terms and variables in equation (2-19) and (2-20) are explicitly defined in the Appendix.

The nature of the response $R(t)$ is derived from that of the systems characteristics \bar{G}_i , δ_i , and η . If these represent the shear stress at the middle of the base layer resulting from a static

load; so does $R(t)$ for random load and temperature histories.

Therefore, $R(t)$ can be any of the time stress, strain, or deformation components σ_{ij} , ϵ_{ij} , and u_i at any point within the system depending on the response function to a step load used as an input to equations (2-19) or (2-20) above.

II.3.4 Probabilistic Analysis - Distress Indicators:

Through the assumptions listed in Section II.3.2 regarding the output variables, one can state the response $R(t)$ in terms of the damage indicators mentioned above, as described below:

1. Rut Depth: This component is obtained through equations (2.19) and (2.20) with a rate of traffic load λ' described as follows:

$$\lambda' = \lambda_c - \frac{\lambda - \lambda_c}{N} \quad (2-21)$$

where: λ_c is the proportion of channelized traffic in one lane.

λ is the total mean rate of traffic in the lane.

N is the number of possible combination of load channels in lane in which the traffic passes (degrees of freedom).

If, for example, 70% of the traffic is channelized at the center of the lane (i.e., $\lambda_c = 0.7\lambda$), and there are three other possible paths that the traffic passes through in one lane of the pavement, then

$$\lambda' = 0.7\lambda - \frac{\lambda - 0.7\lambda}{3} = 0.6\lambda$$

The values of \bar{G}_1 , δ_1 , and η are obtained by simulation of the vertical deflections at the surface of the pavement beneath the center of a step (static) load.

These values with λ' substituting for λ in equations (2-19) and (2-20) yield the means and variances of the rut depth versus time.

2. Slope Variance: In the following analysis the spatial autocorrelation function of the surface deformation is obtained. From which, the slope variance can easily be obtained. The detail of the mathematical work is shown in the second part of Appendix I.

The spatial autocorrelation function of a system's response $\tilde{R}_t(x)$ may be expressed as:

$$\tilde{R}_t(x) = E_x[R(t, x_0)R(t, x_\ell)] - E[R(t, x_0)] E[R(t, x_\ell)] \quad (2-22)$$

where $E_x[\quad]$ signifies that the expectation operation is taken over the space variable x , only and $R(t, x_j)$ is the response of the system at time t and location x_j .

In this case, the response represents the vertical deflection at the surface of the pavement measured at the center of the load as in the case of rut-depth measurement.

Equation (2-22) may be expanded as:

$$\begin{aligned} \tilde{R}_t(x) = E_x \left[\left\{ \sum_{k=1}^L R_k(t, x_0) \right\} \left\{ R_k(t, x_\ell) \right\} \right. \\ \left. - E_x \left[\sum_{k=1}^L R_k(t, x_0) \right] E_x \left[\sum_{k=1}^L R_k(t, x_\ell) \right] \right] \end{aligned} \quad (2-23)$$

Since the roughness (expressed in terms of the spatial autocorrelation function) is a function of the spatial variation ϕ in the materials properties of the system, the only space variables in equation (2-23) will be η , which in this case can be written as η_{x_0} and η_{x_ℓ} or simply η_0 and η_1 , related to points x_0 and x_ℓ respectively.

The analysis in Appendix I results in the following expression:

$$\tilde{R}_t(x) = [\rho_{x_0, x_\ell} \sigma_\eta^2 + 1] \left(\sum_{k=1}^L z_k \right)^2 \quad (2-24)$$

where

$$\rho_{x_0, x_\ell} = \frac{\text{Cov}[\eta_0 \eta_\ell]}{\sigma_\eta^2} \quad (2-25)$$

is the spatial correlation coefficient of the surface deflection in the pavement. The coefficient may be represented by the following expression:

$$\rho_{x_0, x_\ell} = A + B \exp [- |x|^2 / C^2] \quad (2-26)$$

where $|x| = |x_0 - x_\ell|$ is the absolute distance between the two points = x

A is the minimum correlation between points far apart from each other. It may be compared with the endurance limit in fatigue curves.

and B and C are materials properties.

It should be noticed that $A + B = 1$.

If equation (2-26) is substituted in equation (2-24), we get

$$\begin{aligned} \tilde{R}_t(x) = & \left[\left\{ A + B \exp\left(-\frac{|x|^2}{C^2}\right) \right\} \sigma_\eta^2 + 1 \right] \left(\sum_{k=1}^L Z_k \right)^2 \\ & - E[R(t, x_0)] E[R(t, x_0)] \end{aligned} \quad (2-27)$$

Define $\dot{Z}(t)$ as the first space-derivative of the function $Z(t)$, i.e., $\frac{\partial}{\partial x} [\dot{Z}(t)]$, and if $S(t)$ represents the vertical surface deformation, then $S(t)$ will be $\frac{\partial}{\partial x} [Z(t)]$, which is the slope of the surface as a function of time. Since:

$$\sigma_{\dot{s}(t)}^2 = \left. \frac{\partial^2 \tilde{R}_t(x)}{\partial x^2} \right|_{|x| = 0} \quad (2-28)$$

where $\sigma_{\dot{s}(t)}^2$ is the variance of the slope $S(t)$, i.e., the slope variance as defined by the AASHO Road Test, and

$$\left. \frac{\partial^2 \tilde{R}_t(x)}{\partial x^2} \right|_{|x| = 0}$$

represents the second space derivative of the autocorrelation function evaluated at $x = 0$

Equations (2-27) and (2-28) yield:

$$\text{Slope Variance} = \sigma_{\dot{s}(t)}^2 = \frac{2B}{C^2} \left(\sum_{k=1}^L Z_k \right)^2 \sigma_\eta^2 \quad (2-29)$$

where \dot{Z}_k is defined in Appendix I.

Since $\sigma_{\dot{s}(t)}^2$ is a random variable in time because of the randomness in the load history, one can find the expected values and variances of this variable versus time as shown in the Appendix.

These are expressed as:

$$E[\sigma_{\dot{s}(t)}^2] = \frac{2B\sigma_n^2}{C^2} \beta_L^2 \left[\sum_{k=1}^L \{ \lambda (\sigma_A^2 + \bar{A}^2) (I_k^1 + I_k^2) + \lambda^2 \bar{A}^2 I_k^3 \} + 2 \sum_{k=1}^L \sum_{m=k+1}^L \text{Cov}(Z_k, Z_m) \right] \quad (2-30)$$

$$\text{Var} [\sigma_{\dot{s}(t)}^2] = \left(\frac{2B}{C^2} \right)^2 \beta_L^4 \sum_{k=1}^L \text{Var} [Z_k^2(t)] \quad (2-31)$$

The terms in equations (2-30) and (2-31) are defined in the appendix.

3. Cracking: A phenomenological model is used for the prediction of the extent of cracking in the pavement structure. This is based on Miner's hypothesis for damage of materials. The criterion for cracking used in this study is based on fatigue resulting from the tensile strain at the bottom of the surface layer.

This requires the determination of the moments for the radial strain amplitudes at the bottom of the surface layer, using the radial strains obtained for step functions from the static load

program. These moments for the strain amplitudes may be determined from the following equations*:

$$\Delta\epsilon_M = \frac{1}{2} A_\eta \sum_{i=1}^N \frac{\bar{G}_i [1 + \exp\{\frac{-\gamma_M \delta_i \bar{D}}{2}\}]}{[1 + \{\frac{\gamma_M \delta_i \bar{D}}{2\pi}\}]} \quad (2-32)$$

where $\Delta\epsilon_M$ represents the radial (tensile) strain amplitude.

The mean and variance of $\Delta\epsilon_M$ have been obtained by the probabilistic analysis in Appendix I, and can be written as:

$$\begin{aligned} E[\Delta\epsilon_M] = \bar{\Delta\epsilon}_M = & \frac{1}{2} \bar{A} \sum_{i=1}^N \frac{\bar{G}_i [1 + \exp(-\gamma_M \delta_i \bar{D}/2)]}{(\frac{\gamma_M \delta_i \bar{D}}{2\pi})} \\ & + \frac{1}{4} \bar{A} \sigma_D^2 \{ \sum_{i=1}^N \bar{G}_i (\frac{\gamma_M \delta_i}{2}) [\frac{\exp(-\gamma_M \delta_i \bar{D}/2) (1 + \frac{\gamma_M \delta_i}{\pi^2})}{1 + (\frac{\gamma_M \delta_i \bar{D}}{2\pi})^2} \\ & - \frac{2}{\pi^2} \frac{1 + (1 - \gamma_M \delta_i \bar{D}/2) \exp(-\gamma_M \delta_i \bar{D}/2)}{\{1 + (\frac{\gamma_M \delta_i \bar{D}}{2\pi})^2\}} \\ & + (\frac{\gamma_M \delta_i \bar{D}}{2\pi^2})^2 \frac{(1 + \exp(-\gamma_M \delta_i \bar{D}/2))}{\{1 + (\frac{\gamma_M \delta_i \bar{D}}{2\pi})^3\}} \end{aligned} \quad (2-33)$$

*The third part of Appendix I contains the details of this analysis.

$$\begin{aligned}
\text{Var}[\Delta \epsilon_M] = \sigma_{\Delta \epsilon_M}^2 &= \frac{1}{4} \sigma_A^2 \sum_{i=1}^N \frac{G_i [1 + \exp(-\gamma_M \delta_i \bar{D}/2)]}{[1 + (\frac{\gamma_M \delta_i \bar{D}}{2\pi})^2]} \\
&+ \frac{1}{4} \bar{A}^2 \sigma_\eta^2 \left(\sum_{i=1}^N \frac{G_i [1 + \exp(-\gamma_M \delta_i \bar{D}/2)]}{[1 + (\frac{\gamma_M \delta_i \bar{D}}{2\pi})^2]} \right. \\
&+ \frac{1}{4} \sigma_D^2 \bar{A}^2 \left(\sum_{i=1}^N \bar{G}_i \frac{-\frac{\gamma_M \delta_i}{2} \exp(-\frac{\gamma_M \delta_i \bar{D}}{2})}{1 + (\frac{\gamma_M \delta_i \bar{D}}{2\pi})^2} \right. \\
&\left. \left. - 2 \frac{[1 + \exp(-\gamma_M \delta_i \bar{D}/2)] (\frac{\gamma_M \delta_i}{2\pi})^2 \bar{D}}{[1 + (\frac{\gamma_M \delta_i \bar{D}}{2\pi})^2]} \right) \right) \quad (2-34)
\end{aligned}$$

Miner's law can be expressed as:

$$D(t) = \sum_{k=1}^L \frac{n_k}{N_k} \quad (2-35)$$

Where $D(t)$ is the damage at time t , resulting from a repetition of loads over L periods of time.

N_k represents the number of loads to failure at the k^{th} period, having the same statistical properties as the n_k loads. The ratio n_k/N_k represents the proportion of damage in terms of fatigue cracking in the k^{th} period.

A fatigue law has been used to determine N_k , the number of loads to failure in the k^{th} period in terms of the tensile strain amplitudes obtained above.

$$N_k = C(T) \left(\frac{1}{\Delta \epsilon_k} \right)^{a(T)} \quad (2-36)$$

where (c) and (a) are material characteristics of certain statistical properties, which are temperature dependents.

Appendix I, under the subheading "Probabilistic Formulation of Minor's Law" presents the analysis to obtain the expected value and variance of N_k , and eventually, the expected value and variance of the damage $D(t)$ versus time. These are:

$$E[D(t)] = \sum_{k=1}^L \left\{ \frac{\bar{n}_k}{\bar{N}_k} + \frac{n_k}{\bar{N}_k^3} \sigma_{N_k}^2 \right\} \quad (2-37)$$

where n_k is the mean number of Poisson loads in the k^{th} period.

\bar{N}_k is the mean number of loads to failure in the k^{th} period and $\sigma_{N_k}^2$ is the corresponding variance.

$$\text{Var } [D(t)] = \sum_{k=1}^L \left[\frac{\sigma_{n_k}^2}{\bar{N}_k^2} + \left(\frac{\bar{n}_k}{\bar{N}_k^2} \right)^2 \sigma_{N_k}^2 \right] \quad (2-38)$$

where $\sigma_{n_k}^2$ is the variance of traffic loads in the k^{th} period = n_k for a Poisson process.

The above damage indicators have been expressed in algorithmic forms and are obtained readily by computer analysis to be used in the next step in the system of hierarchy of the present analysis, namely to determine the serviceability of the system with time and the associated reliability and life expectation.

II.4 Serviceability--Maintenance Model

II.4.1 The Problem of Service Evaluation

Traditionally, the services provided by a highway pavement have been tacitly described in terms of some manifestations of deformation and disintegration. These manifestations have often been arbitrary as to the limits they impose upon what constitutes satisfactory service, and have been developed usually from empirical and rule-of-thumb practices. Most of the present pavement design practices, for example, take into consideration the load bearing characteristics of the pavement in terms of the maximum allowable stress or deformation as the only design criterion. However, it is widely recognized that surface characteristics of the pavement have important effects upon the road's adequacy as a transportation link in terms of safety and ride, which are not accounted for in these design methods.

The AASHO Road Test provided one of the early recognitions that a pavement is providing a transportation service and must be analyzed in broader terms. The adoption of the term "present serviceability" to represent the ability of a pavement to serve traffic,

and the interpretation of service as deriving from users' response represented a break with previous practice. Yet even in its formulation, the AASHO approach is restricted to consideration of pavement surface riding quality. It neglects such factors as maintenance and safety features of the road.

The user of a highway as the recipient of the benefits of transportation, provides a link between the highway and the social, political, and economic systems which the highway serves. The pavement must be viewed as a part of a transportation system, and if the pavement is to fulfill this role, it must provide service to the user. That is the evaluation of a pavement's physical behavior must be made in terms of users' wants and needs.

II.4.1.1 Measures of Effectiveness

It has been suggested that the term serviceability may be defined as the degree to which adequate service is provided to the user, from the user's point of view (L1, L2). Implementation of this concept as a measure of effectiveness for decision making provides a translation of the requirements of the larger role of the pavement into terms of physical importance. For pavements, serviceability is represented by three components: rideability, safety, and structural integrity.

Rideability refers to the quality of ride provided by the pavement, and is measured by the users' response. Included are such factors as comfort and likelihood of damage to goods. Safety depends upon skidding characteristics of the pavement and the presence of such things as obstructions and glare spots which might

increase the likelihood of accidents. Structural integrity refers to the ability of the pavement to meet future load demands for which it was initially designed.

Evaluation of service in these terms requires consideration of many factors which are largely subjective, comprising the users' perceptions of a pavement's behavior. The basic assumptions which have been used to permit a uniform approach to evaluation is that it is possible to identify a level of behavior which will be judged to be at least adequate by the individual user, and that the overall evaluation of service may then be made in terms of the probability that a particular level of service will be so judged by a user. Using arguments based upon utility theories (R1, P3) as presented in economics and psychology, it may be suggested that these assumptions are valid for the evaluation of pavement service. There then remains the problem of determining the serviceability of a pavement exhibiting a particular physical behavior.

Using such methods as utility theory, one may assess the users' evaluation of service delivered by the highway pavement. Through the development of causal and statistical prediction models, one then can have the ability to determine the user's assessment from measurable physical characteristics of the pavement structure. In a somewhat limited way, this is what was achieved in the AASHO Road Test, when evaluations of riding quality were statistically related to the surface deformation and the areas of cracking and patching in the pavement.

II.4.2 Probabilistic Manipulation of AASHO's Present Serviceability Index:

The present serviceability index can be expressed as a function of some objective distress components, say cracking and deformation.

Using the same components used by AASHO, one can write a general expression for the present serviceability index as:

$$PSI = f[RD, (C + P), SV] \quad (2-39)$$

where RD refers to rut depth.

C + P refers to area cracking and patching.

SV refers to the slope variance.

A linear combination of function of these components can be a possible form for equation (2-39).

$$PSI = A + B(RD) + C(C + P) + D(SV) \quad (2-40)$$

In the present study, AASHO's present serviceability expression has been used without change. However, it is possible to use alternative forms which may include other variables that may be deemed as necessary for evaluation of a system's serviceability.

AASHO's equation is a special form of equation (2-40) and can be written as:

$$PSI = C_1 + C_2 \log(1 + SV) + C_3 (\sqrt{C + P}) + C_4 (RD)^2 \quad (2-41)$$

where:

$$\begin{aligned} c_1 &= 5.03 & c_2 &= -1.91 \\ c_3 &= 0.01 & c_4 &= -1.38 \end{aligned}$$

If we treat the components SV, C + P, and RD as random variables, the PSI will be a random variable, too, and we can determine the moments of PSI at any instant k.

Appendix I presents an approximate probabilistic analysis for determination of the moments of PSI(k), assuming that SV, C + P, and RD are independent components.* These moments can be written as:

$$\begin{aligned} E[PSI(k)] &= c_1 + c_2 \left\{ \text{Log} (1 + \overline{SV}_k) - \frac{1}{2} (1 + \overline{SV}_k)^2 \sigma_{SV_k}^2 \right\} \\ &\quad + c_3 \left\{ \overline{D}_k - \frac{1}{8} \overline{D}_k^2 \sigma_{D_k}^2 \right\} + c_4 \left\{ \overline{RD}_k^2 + \sigma_{RD_k}^2 \right\} \quad (2-42) \end{aligned}$$

$$\begin{aligned} \text{Var}[PSI(k)] &= c_2^2 \frac{\sigma_{SV_k}^2}{(1 + \overline{SV}_k)^2} + \frac{1}{4} c_3^2 \frac{\sigma_{D_k}^2}{\overline{D}_k} \\ &\quad + 4c_4^2 \overline{RD}_k^2 \sigma_{RD_k}^2 \quad (2-43) \end{aligned}$$

*Independence implies that knowledge about one variable does not enhance our knowledge about the other variable, at least significantly. This can be realistic to some extent in this case in the sense that knowledge about cracking does not tell us much about the quantitative values of roughness or rutting. Similarly, rutting may not provide clear information about roughness or cracking, and so on.

where

\overline{SV}_k , \overline{D}_k , \overline{RD}_k refer to the expected values of slope variance, cracking, and rut depth respectively at the k^{th} period.

and

$\sigma_{sv_k}^2$, $\sigma_{D_k}^2$, $\sigma_{RD_k}^2$ represent the corresponding variances.

In order for the serviceability to be used in any meaningful way, it is necessary to establish some probability distribution function in which values of the serviceability and their associated probabilities are evaluated at any time.

Since the serviceability is expressed as a sum of damage components, and since these components have been assumed as independent in this study, one can invoke the central limit theorem, to state that the distribution of the serviceability index is nearly Gaussian. The central limit theorem, in effect, states that "a random variable which is a sum of a number of random variables tends to approach a normal distribution as the number of random variables in the sum becomes sufficiently large [B1, F1, P1]. This theorem has been proved under a variety of conditions has been provided by Parzen (P1) and Feller (F2), and some others. Some of the necessary conditions for this proof are satisfied by the serviceability equation. These necessary conditions state that the variance of the process should be limited and that there must be a physical meaningfulness for the process. In the case of serviceability, the physical interpretation of the index in terms of the objective damage measures is apparent. The variance of PSI is limited since

the system can only be in a range of values of PSI which are bound by the physical characteristics of the system.

The probability density function for a normal distribution may be expressed as:

$$f_S(s) = \frac{1}{\sqrt{2\pi} \sigma_S} \exp \left[-\frac{1}{2} \left(\frac{s - \bar{S}}{\sigma_S} \right)^2 \right] \quad (2-44)$$

Defining: $U = \frac{s - \bar{S}}{\sigma_S}$ (2-45)

U is called the standardized normal random variable which has a (0, 1) distribution.*

$$f_U(u) = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} u^2 \right] \quad (2-46)$$

and

$$f_S(s) = \frac{1}{\sigma_S} f_U \left(\frac{s - \bar{S}}{\sigma_S} \right) \quad (2-47)$$

The cumulative density function may be defined as:

$$F_S(s) = P[S \leq s] = P[U \leq \frac{s - \bar{S}}{\sigma_S}] \quad (2-48)$$

$$F_S(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u \exp \left(-\frac{1}{2} u^2 \right) du \quad (2-49)$$

*Mean = 0, and Variance = 1.0

If one is interested in determining the probability, the present serviceability index lies between any two values S_1 and S_2 , where $S_2 > S_1$, then one has to obtain the cumulative function at both values, i.e.,

$$F_S^2(s) = P[S \leq s_2] = F_U \left[\frac{s_2 - \bar{S}}{\sigma_S} \right]$$

$$F_S^2(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{s_2 - \bar{S}}{\sigma_S}} \exp \left(-\frac{1}{2} u^2 \right) du \quad (2-50)$$

and

$$F_S^1(s) = P[S \leq s] = F_U \left(\frac{s_1 - \bar{S}}{\sigma_S} \right) \quad (2-51)$$

$$F_S^1(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{s_1 - \bar{S}}{\sigma_S}} \exp \left(-\frac{1}{2} u^2 \right) du$$

then:

$$q_1(t) = P[S_1 \leq s \leq S_2] = F_S^2(s) - F_S^1(s) \quad (2-52)$$

Equation (2-51) above defines the marginal probability for the serviceability at any point in time t , where i refers to some level of serviceability that can be arbitrarily defined between the limits S_1 and S_2 .

The determination of the marginal, or state, probabilities for the serviceability index is very useful in predicting the life

expectancy and the distribution of life of the system as well as the reliability of the system at any time. These probabilities are also extremely useful in any decision structure for maintenance practices for the system under consideration, as will be explained in the following chapter.

II.4.3 Markovian Behavioral Model for Pavement Performance

II.4.3.1 Introduction: Markov Processes and Physical Interpretation

Highway pavements are very complex systems having highly unpredictable behavior and performance. This is a result of the complex physical process of deterioration in an uncertain operational environment, and of the periodical maintenance activities experienced on the system.

A Markovian approach of modelling has been used in this study to define the deterioration law for pavement systems. A Markov process is one in which the future course of the system is only dependent on its present state and not on its entire past history. Mathematically stated, a discrete parameter stochastic process $\{X(t), t = 0, 1, \dots\}$ is said to be a Markov process if, for any set of n time points $t_1 < t_2 < \dots < t_n$ in the index set of the process and any real numbers x_1, x_2

$$\begin{aligned}
 P[X(t_{n+1}) < x_{n+1} \mid X(t_1) = x_1 \cap \dots \cap X(t_n) = x_n] \\
 = P[X(t_{n+1}) < x_{n+1} \mid X(t_n) = x_n]
 \end{aligned}
 \tag{2-53}$$

Intuitively, equation (2.53) implies a one step dependence: given the present, the future states are independent of the past.

The use of a Markov model to describe the damage or serviceability of pavement systems has been motivated by the following rationale:

- i. One can assume that an approximate exponential law defines the progression of cracks and accumulation of deformation in pavement structures.

This means that the individual damage components defining the serviceability of the system are approximately defined by an exponential growth relation. A Markov law is generally suggested to define the overall process because this is the type of process a Markov property may approximately represent (B2).

- ii. A Markov process is the stochastic equivalent of the type of process in physical systems whose first order description is that in which knowledge of history of the system yields no predictive value (B2). To illustrate this point, consider a highway pavement under operational conditions. Damage initiates and propagates in the pavement structure as a result of traffic loads and environmental effects. Materials properties are inherently variable, so are the characteristics associated with load and environment. Maintenance is practiced on the system on different levels at different times depending on the

the degree of damage and on the constraints of performance, reliability, and cost of construction and operation. Therefore, it is extremely difficult to obtain precise predictive information about the serviceability of the system at any time. Certain operational strategies may be needed momentarily to account for unpredictable changes in damage progression as a result of sudden changes in patterns of traffic and environment. Markov processes are stochastically equivalent to this type of behavior, and have been used accordingly in this study.

- iii. The behavior of the materials within the pavement layered structure is generally time dependent. This has been represented by a linear viscoelastic model whose behavior is represented by a hereditary integral. In this class of materials, there is a restricting principal which is the principal of Fading Memory, which postulates that the influence of heredity related to past states gradually fades out with time [W1, C2, G1]. What this principal essentially says, is that at some point in time, it is not important to know the whole history of the system in order to predict its future behavior. Therefore, given the knowledge about the present we can project future states. The definition of Markov transition says the same thing in probabilistic terms. The question that

remains in the adoption of a Markov model for pavement materials is that related to time spacing as to what constitutes "sufficient" period for fading of the memory. Obviously, this depends on the nature of the materials under consideration and may be effectively answered by characterization of these materials. This also depends on the level of accuracy required and the significance of errors involved in disregarding past history that may affect the future behavior to a small extent. In this study periods of at least one month are being used and it is reasonable to argue that the response to a load applied ten minutes from some present observation depends only on the present period and not one month before this period.

II.4.3.2 Serviceability Representation in Markov Chains

The levels of serviceability of the system as predicted by AASHO's representation can be used to define stages or states in the Markov chain. This requires the discretization of the otherwise continuous state-time process by dividing the states space into any desirable number of levels. Each level is bounded by upper and lower values of serviceability.

The transition from one state to another implies a change in the level of serviceability due to the accumulation of damage which is a temporal process, or due to improvement of the state of the system which essentially involves no significant time lapse.

In general, a continuous process of transitions from one state to another starts from some initial state. This initial state defines the quality of the system at the outset of its operation. The characteristics of state transition depend on the properties of the system, its inputs, and on the maintenance activities experienced throughout the life time of the system. The characteristics are defined by a transition probability matrix $[p_{ij}^{(t)}]$ and an associated reward matrix $[r_{ij}^{(t)}]$. The transition matrix defines the probabilities of changing from one serviceability level to another in a certain period of time. The amounts of the change associated with state transitions during these time periods are described by the reward matrix.

A schematic representation of a four-state Markov chain is shown in Figure 6. The arrows show the directional state transitions with probabilities of such transitions. State four is the final state where failure is assumed to have occurred. This state is often referred to as the trapping state, which is the state that once the system enters, it stays in. A subjectivistic interpretation of the trapping state may be that at this state the level of serviceability, measured by some objective indicators in the case of AASHO, is perceived as unacceptable by the users of the facility. The level of maintenance required to improve the system at this stage becomes high enough that a total resurfacing is needed. This point may be arbitrarily chosen as the life of the pavement. From figure (6), one can define the one-step

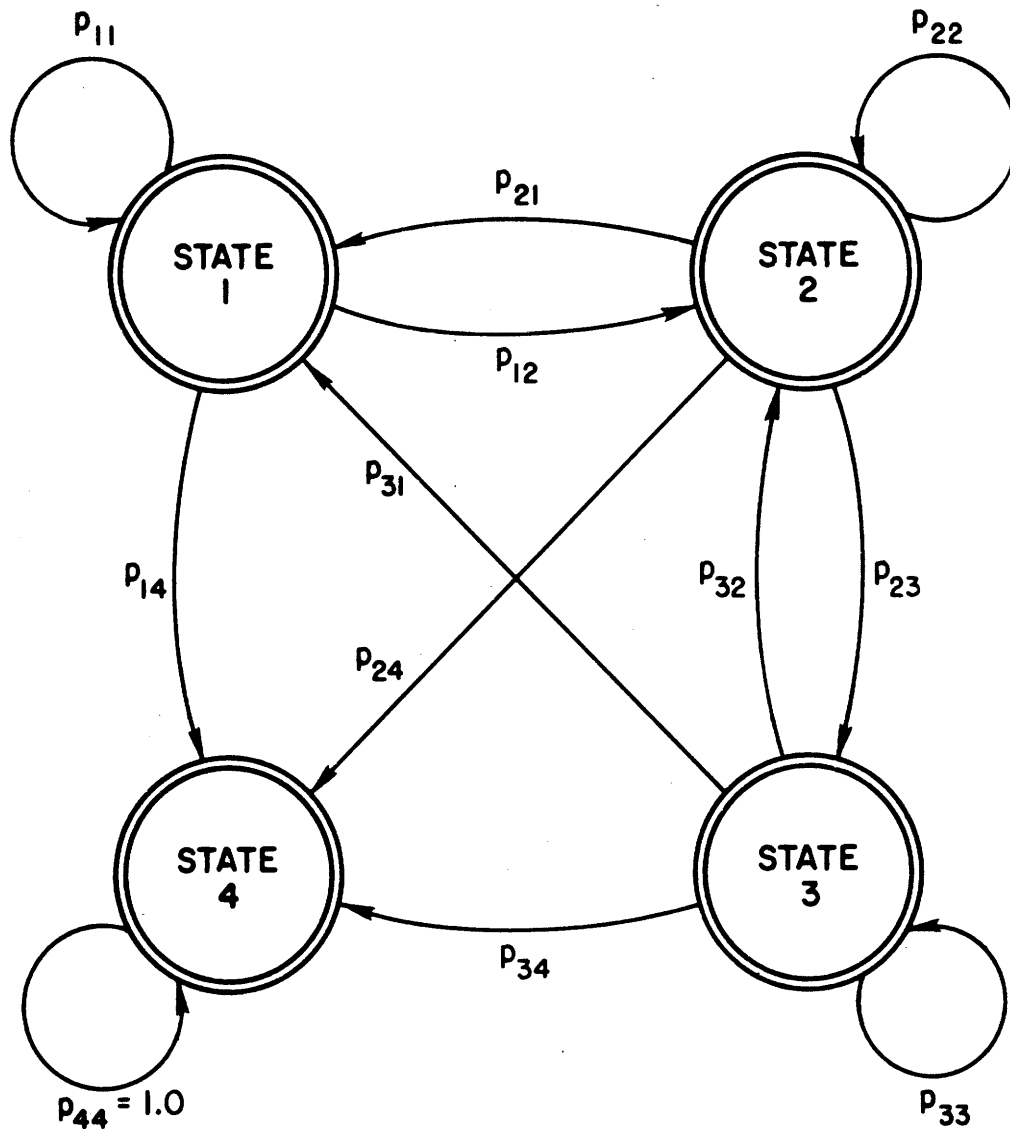


FIGURE 6. A FOUR-STAGE TRANSITION MARKOV CHAIN.

transition* matrix P as follows:

$$\bar{P} = [P_{ij}] = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \quad (2-54)$$

If no maintenance takes place, the transition matrix reduces to an upper triangular matrix of the form:

$$\bar{P} = [P_{ij}] = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ 0 & P_{22} & P_{23} & P_{24} \\ 0 & 0 & P_{33} & P_{34} \\ 0 & 0 & 0 & P_{44} \end{bmatrix} \quad (2-55)$$

One can extend this example to the system at hand. To do so, we must define the Markov states in terms of the serviceability level of the system. This requires the discretization of the serviceability index into a number of levels, and defining upper and lower bounds for these levels by assigning numerical values to them. Each level is referred to as a state in the model. Once this has been accomplished, it is possible to assign probabilities to these states at any time period using the information obtained

*One-step transition means that the transition for one state to another in a unit time period, e.g. one month, three months, six months, one year, etc. All transitions occur between periods n and $n + 1$.

from the previous sections. It is more difficult, however, to obtain the transition probabilities or the transition matrices due to the lack of information from field and experimental observations in this respect. The next section deals with the methodologies used in probability assignments utilizing the framework established above.

II.4.3.3 Marginal State and Transition Probability Assignments

A. Marginal State Probabilities

Analysis based on expected values and averages yields no indication of probabilities of premature failures or unexpectedly long life periods, nor does it provide ample information for decision making criteria for selection of alternatives on this basis. Probabilities of failure and/or success are of extreme importance in considering maintenance policies or quality control in system fabrication. Since the first depends upon the m -step transition probabilities, and the latter on the marginal state probabilities, it is essential to determine these probabilities.

Marginal probabilities refer to the probabilities of being in any state i , i.e. level of serviceability corresponding to that state, at any time period t . These are obtained from the probability distribution of the serviceability index at the desired time period. For example, if at time t_1 one wishes to find the probability of being in state j , which has an upper bound value of serviceability of x_1 and a lower one of x_2 , then one must find the cumulative distribution function at these two points.

Therefore:

$$\frac{x_1 - X(t_1)}{\sigma_X(t_1)}$$

$${}_1F_X^{t_1}(x) = P[X \leq x_1] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{x_1 - X(t_1)}{\sigma_X(t_1)}} \exp\left(-\frac{1}{2} u^2\right) du \quad (2-56)$$

$$\frac{x_2 - X(t_1)}{\sigma_X(t_1)}$$

$${}_2F_X^{t_1}(x) = P[X \leq x_2] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{x_2 - X(t_1)}{\sigma_X(t_1)}} \exp\left(-\frac{1}{2} u^2\right) du \quad (2-57)$$

$$P[x_1 \leq X_{t_1} \leq x_2] = {}_1F_X^{t_1}(x) - {}_2F_X^{t_1}(x) \quad (2-58)$$

where:

${}_iF_X^{t_1}(x)$ denotes the value of the cumulative function of x_i at t_1 .

Equation (2-58) yields the required marginal state probability, i.e., the probability of being in state j at time t_1 . The above expressions have also been discussed earlier in Chapter 2.

B. State Transition Probabilities

The m -step transition probability $P_{ij}^{(m)}$ denotes the probability that the system changes its serviceability level from state i to state j in m periods of time.

The generation of the P_{ij} matrices is a process which greatly depends upon the type of Markov chain used. If one selects a non-homogeneous system we have a time dependence of these matrices. If a homogeneous system is chosen, then there is no time dependence and $P_{ij}(n) = P_{ij}(n+1)$ for all n . In any case the m step transition probability is defined as follows:

$$P_{ij}^{(m)}(n) = \sum_{k=1}^r P_{ik}^{(m-1)}(n) P_{kj}(n+m) \quad (2-59)$$

which is the probability of being in state j at time $n+m$ given being in state i at time n , with r being the total number of states.

In dealing with marginal state probabilities, we wish to know the effect of the initial state upon the later states. So we have

$$q_j^{(m)} = \sum_{i=1}^r q_i(0) P_{ij}^{(m)}(0) \quad (2-60)$$

where $q_j^{(m)}$ is our marginal state probability at time m and $q_i(0)$ the initial state probability.

The above equations apply to both homogeneous and non-homogeneous systems, with the difference being in the constancy

of $p_{ij}(n)$ for homogeneous systems.

The generation of a transition matrix is dependent on whether or not the system is homogeneous in time. A discussion on the time-dependency of the transition probabilities is presented below. To simplify the present analysis, a homogeneous process has been considered, with the potentiality of modifying the analysis at any stage to time-dependent (i.e., non-homogeneous) behavior, should the field studies and validations show the need for such calibration.

In non-homogeneous systems, the transition matrices are time-dependent. What this says is that if we are at state 3 after 3 years, then the probability of going to state 4 in the next year is different than the probability of going from state 3 to state 4 after 8 years, for example. In this process, the chain is still memoryless in the sense that it makes no difference what path was followed to get to state 3, but it makes a difference in how long it took to get to that state.

The approach to this problem takes the knowledge that a serviceability function of the form $PSI = f(SV, C+P, RD, t)$ is known whose marginal probabilities of states can be determined at all time periods.

The determination of transition probabilities is based on the assumption that the system is homogeneous in time. For a homogeneous transition chain, equation (2-60) can be expressed as follows:

$$q_i(m) = \sum_{i=1}^r q_i(m-1) p_{ij}^{(1)} \quad (2-61)$$

Since the system does not improve with time, except when maintenance is applied, one can state that:

$$q_j(m) = \sum_{i=1}^j q_i(m-1) p_{ij}^{(1)} \quad (2-61A)$$

To obtain p_{ij} , one has to resort to the multi-stage transition with time as follows:

$$q_j(m+1) = \sum_{i=1}^j q_i(m) p_{ij} \quad (2-61B)$$

$$q_j(m+k) = \sum_{i=1}^j q_i(m+k-1) p_{ij}^{(1)}$$

Equations (2-61) constitute a set of simultaneous linear equations of the form

$$[Q] = [Q'] [P] \quad (2-62)$$

where

$[Q]$ is a column vector of j elements

$[Q']$ is a square matrix of $j \times j$ elements,

and $[P]$ is the transition probability matrix.

If there are n states in which transition occurs, then n sets of equations of the form described by (2-62) are needed to obtain the n - P_{ij} probabilities. These n -solutions yield an upper triangular matrix as described by equation (2-55) above.

When maintenance is applied, some elements of the lower P_{ij} matrix which are zeros are replaced by a value of 1.0 at that time period.

From this, one can find the m -step transition probabilities simply by raising the one-step transition matrix to the m^{th} power.

It must be emphasized that in the determination of transition and marginal probabilities, it is assumed that the system does not improve with time, i.e., $P_{ij} = 0$ for all $i < j$. This implies that no maintenance action has been introduced. Later in this chapter, the influence of maintenance is introduced and the modification of state and transition probabilities due to this factor is discussed.

II.4.4 System's Reliability

II.4.4.1 The Concept of Reliability in Systems

A system's reliability may be defined as the probability of having a satisfactory performance state for the system. Therefore, in order to quantify reliability, the "state" of "satisfactory performance" must be defined and the probability of having such an event must be determined.

Reliability of a system can be characterized by:

- (1) the system's inputs, (2) interaction of the system's characteristics with its inputs, and (3) the responses or outputs of the

system, which represent the entire spectrum of operational states (both satisfactory and unsatisfactory performance).

A more general definition for reliability may, then be stated as "the probability that the system will provide a satisfactory performance for, at least, a given period of time, under a stated set of conditions" (H1). This definition makes some important points. It suggests that reliability is the probability of a success event. This event is defined by the three different conditions mentioned in the above definition.

1. "The system will provide a satisfactory performance which requires.
 - (a) the functional and physical characteristics of the constituents of the system.
 - (b) a definition of the stipulated services or function that the system is to provide, and
 - (c) a definition of failure criteria for the system.
2. "For at least a given period of time", that requires the prediction of the times over which the various components of the system are to be operational.
3. "Under a stated set of conditions", requires the identification of all the load and environmental variables operating in the system, and their properties and distributions.

These three conditions have been defined in the preceeding sections. This section is concerned with the development of reliability measurement based on the behavior of the system discussed earlier.

II.4.4.2 Time Dependent Reliability of Highway Systems

Highway pavements may be identified as structure-sensitive systems. In these systems, failure is not a clear-cut phase that occurs abruptly rendering the system unserviceable. Failure is the extent of progressively developing structural deterioration resulting in the loss of performance. Therefore, the system conceivably starts with a rather high level of performance. As a result of exposure to the environment and load, this level of performance gradually drops down in a manner commensurate with the system's characteristics. This gradual drop may, therefore, be accelerated if certain conditions prevail such as aging etc., or it may be very slow if the properties of the materials and the environment are suitable for this type of action.

What this suggests, then, is that the system starts with high level of reliability, depending on the degree of quality control in building the system that defines its performance level initially, and the reliability of the system drops down as its performance level decreases with time. Therefore, reliability of highway systems is a time-dependent parameter.

Time-dependent reliability will then be concerned with the rate of failure (or damage) production in the system. This rate of failure production may be decreased or increased depending on the rate of damage progression and maintenance introduction into the system.

Several models have been proposed for the distribution of the reliability of the system. These models also lead to the

determination of the distributions of lifetime of the systems. Of those models are: exponential, Gamma, Weibull, Truncated Normal, General life distribution (W2), and so on. It is suggested that a careful consideration should be given to this area, beyond the scope of this study, and it is believed that the choice of a model may lead to the computation of transition probabilities of the Markov process or at least shed light on their order of magnitudes. In the present study, however, the reliability of the system at any time was computed by transition probabilities that have been estimated in the previous section.

II.4.4.3 Numerical Measurements of Reliability

Having defined the systems reliability, it is now possible to examine how our simplified model applies to this definition.

The reliability of the given system can be defined in two ways. The first makes use of the marginal state probabilities and is what would be used by the designer in determining whether or not his system is "good enough" to meet "specifications". The second uses the m-step transition probabilities and is used, most likely, by the maintenance engineer in determining the present reliability of a system at some time interval after construction.

The former would depend upon the quality control of the design and fabrication and will give the system reliability at some time m , based on the probability of being in a failure state j at time m and is defined as $1 - q_j(m)$ where $q_j(m)$ is the

marginal failure state probability at time m and

$$q_j(m) = \sum_{i=1}^r q_i(0) p_{ij}(m) \quad (2-63)$$

for a homogeneous chain. And $q_i(0)$ is the initial state probability or the probability of being in state i upon completion of construction.

For the maintenance engineer the marginal state probabilities are not sufficient and we use the m step transition probabilities to determine the present value of the system reliability, which may be greater or less than the original design reliability. The formula which applies here is:

$$R(n+m) = 1 - p_{ij}^{(m)} = 1 - \sum_{k=1}^i p_{ik}^{(m-1)} p_{kj} \quad (2-64)$$

for a homogeneous chain where m defines the number of steps (years) into the future for which the reliability is considered.

In both cases, it should be noted that the reliability is the probability of not being in the trapping state, which is the failure state.

The approach pursued in the present analysis is as follows: Once the unacceptable level of serviceability s^* (i.e., the trapping state is defined, subjectively or otherwise, then the reliability of the system at time t is defined by the cumulative distribution function $F_s(s^*)$ at that time. The cumulative distribution function defines "the probability that the system will be in state s^* or less."

II.4.5 Life Expectancy and Distribution of Times to Failure:

Within the context of this study, the time to failure is the first passage time at which the system enters the trapping state, i.e., the state of unacceptable serviceability level. At this time resurfacing is presumably required to return the system back into a serviceable or operationable state. Generally, this may be regarded as a maintenance action beyond which the system continues to function in a satisfactory manner. However, the above definition of service life has arbitrarily chosen, and one can use this definition in a more general context as time between resurfacings.

If the serviceability index of the system is divided into n states, then the failure state will be referred to as state n . The random variable T , the time from initial construction until the systems enter state n , will have the following cumulative distribution function:

$$F_T(t) = P[T < t] = q_n(t) \quad (2-65)$$

In which $q_n(t)$ is the marginal state probability, as defined in the previous sections. This is the probability that the system will be in state n at time t , which is equal to $1 - R(t)$; $R(t)$ being the system's reliability at time t . From equation 2-63, the probability mass function* (PMF) of the random variable T can be obtained, by discretizing the time intervals into steps of 1 month, 3 months, 6 months, one year or as desirable. This may be expressed as:

$$P_T(t) = F_T(t) - F_T(t-1) \quad (2-66)$$

*The probability mass function is the discrete counterpart of the (continuous) probability density function.

where $P_T(t)$ is the probability that failure time T is equal to t .

For example, one can obtain the following cumulative function and mass function for an n -state system:

The CMF is:

$$\begin{aligned}
 F_T(1) &= P[T < 1] = q_n(1) \\
 F_T(2) &= P[T < 2] = q_n(2) \\
 F_T(3) &= P[T < 3] = q_n(3) \\
 &\vdots \\
 F_T(m) &= P[T < m] = q_n(m)
 \end{aligned}
 \tag{2-67a}$$

and the PMF is:

$$\begin{aligned}
 P_T(1) &= F_T(1) \\
 P_T(2) &= F_T(2) - F_T(1) \\
 P_T(3) &= F_T(3) - F_T(2) \\
 &\vdots \\
 P_T(m) &= F_T(m) - F_T(m-1)
 \end{aligned}
 \tag{2-67b}$$

The expected value of life may be obtained from equations (2-67) above utilizing the definition for the probability mass function; for life L :

$$E[L] = \sum_i t_i P_T(t_i)
 \tag{2-68}$$

in which $t_i = 1, 2, 3, \dots$ and $p_T(t_i)$'s are obtained in equation (3.16) above.

II.4.5.1 Distribution of Times to a Given State

The results obtained from equations (2-65) and (2-66) are true when state n is defined to be the failure or trapping state. However, one can obtain the distribution of the time to first passage into any desired state, i . This can be accomplished by making state i a trapping state (with $P_{ii} = 1$) and making use of the modified transition probabilities to determine the modified state probabilities using equation (2-61) for example. The new state probabilities are interpreted as the cumulative distribution function for the time to the first entrance into that state (B1). Accordingly, one can get the PMF and first time passage probabilities for any state i in a manner to that explained earlier in this section.

The discussion above applies in the absence of any maintenance activity to improve the state of the system and modify the associated state and transition probabilities. As maintenance is applied to the system, the procedure for determining life distribution is exactly similar to that discussed above, except that the state and transition probabilities are modified to incorporate effects of the maintenance. These and other issues related to maintenance will be discussed in the next section, and a framework for a decision structure to achieve an optimum maintenance strategy for a given system will be discussed in Chapter IV.

II.4.6 Maintenance Activities and Modification of the Markov Model

Maintenance of a system may be viewed as the work performed on the system during its operational life to improve it, assure a "desirable"

serviceability level, improve its life expectancy at a desired level of reliability.

Quantitatively, maintenance may be viewed in the same manner as the serviceability. In this respect, maintenance may be regarded as an algebraically negative damage. Maintenance activities at each point may be defined as a temporal vector of a quantity and an associated cost. The temporal nature of the maintenance vector is introduced through a discounting factor of the accrued cost at different times to scale all cost factors to a single reference point such as initial time or any other time reference.

In order to introduce the factor of maintenance into the model and to establish a framework for cost-benefit analysis for the system, the concept of reward structures is introduced. As the system makes a transition from state i to state j , an associated reward, in terms of cost or benefit, results from that transition. This cost or benefit may be measured in terms of dollars lost or gained, performance increased or decreased, reliability, safety, or any other measurable physical quantity that is pertinent to the problem at hand. In certain cases, the reward may be a vector of a set of attributes, a particular combination of which serves as a yardstick for evaluation.

In this section, the reward is presented in terms of a reward matrix defining the loss or gain of serviceability resulting from state transitions. Each element in the reward matrix has an

associated transition probability as defined in the previous sections. Therefore, for the four-state transition shown in figure (6) and discussed earlier, one can define a reward matrix of the following form:

$$\bar{R} = [r_{ij}] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix} \quad (2-69)$$

In which r_{ij} is the "amount" of serviceability lost or gained as the system makes transition from state i to state j in time t with a probability P_{ij} . One uses the convention that a positive value of r_{ij} denotes a gain in serviceability and a negative value is the result of a loss in serviceability of the system. In the above equation, all rewards r_{ij} in which $i < j$, i.e., where the system "jumps" into a state of lower level of serviceability, have negative values referring to the loss in the level of serviceability. The transition $i < j$ is a temporal process associated with deterioration resulting from action of loads and environment. On the other hand, all rewards r_{ij} in which $i > j$ are algebraically positive denoting an improvement in the state of the system as a result of maintenance activities. There are two interesting properties for those reward vectors that have been assumed in this study. One is that the transition occurs in t step-like, i.e., there is no time lapse involved in such a

transition.* The other property is that the probability P_{ij} associated with a transition from state i to state j due to maintenance takes the form of a kronecker delta function: if maintenance takes place, then $P_{ij} = 1$, otherwise it is equal to zero. This provides the numerical "filler" for the lower triangle in the transition matrix of equation (2-55). Mathematically this may be expressed as:

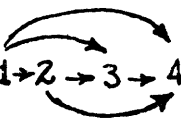
$$P_{ij} = \delta_{ij} = \begin{cases} 1 & \text{with maintenance} \\ 0 & \text{with no maintenance} \end{cases} \quad \text{for all } i < j \quad (2-70)$$

To illustrate how maintenance activities enter into the scope of this study, the four-step transition example in figure (9) is referred to again.

At the outset, the system can be in any of the four states, but for simplicity let us assume that the quality control was so designed that the system can only start at state 1 or at state 2 with probabilities P_1 and P_2 , respectively. As the system is subjected to external perturbations, it will make jumps from one state to another until it reaches state 4 where it stays indefinitely; in other words, state 4 is the failure state. If no upgrading is involved during the operation of the system, the state transition is only unidirectional, or at least non-multidirectional**, i.e.,

*It is assumed that maintenance activities are exercised almost instantaneously such that time between transition due to these activities may be regarded as zero.

**Since it is possible for the system to stay in its current state, the transition is only within the same state, but the system can never move to a better state.



the state transition can only be $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$. This means that all transition probabilities P_{ij} , where $i > j$ in this example, are zero. However, when maintenance is introduced, then the transition is not any more unidirectional but can be forced to change course at the influence of maintenance. One can determine quantitatively the increase in the serviceability of the system for a certain level of maintenance. This may follow the same procedure which was used in the case of serviceability. Since serviceability can be related to damage regressively, and since maintenance activities affect the level of damage, then it is possible to use the serviceability measure for maintenance and scale it accordingly. Therefore, if at any time t , the system is in state i with probability P_i , and if maintenance was introduced, then the system will make an upward transition to state j with probability $P_{ij} = 1.0$. The increase in serviceability due to this transition will be expressed in terms of a reward vector r_{ij} , and there may be a cost of maintenance associated with such transition, say c_{ij} . This cost may be scaled down to some instant in time through a discounting factor such that all costs accrued from maintenance are measured in homogeneous manner. The use of a net-present value criterion is a suitable example for this type of scaling.

Once the transition probability and reward* matrices are constructed, their product yields an expected measure of serviceability, expected cost, or some other attribute throughout the

*Reward expressed in terms of serviceability loss or gain, cost, or any other attribute or combination of attributes. Alternative forms of this function are discussed in the next chapter.

period of study. This is shown in the following expression:

$$E[S] = [P_{ij}]^n [r_{ij}] \quad (2-71)$$

where

$E[s]$ is the resulting expected measure

$[P_{ij}]^n$ is the n-step transition probability matrix, and

$[r_{ij}]$ is the associated reward matrix.

From the analysis presented above, one can observe how a particular design configuration and a certain combination of maintenance activities are combined together to achieve a certain level of serviceability at some cost. Moreover, one can find out how maintenance can be applied* to achieve a certain level of design requirements at some cost to be determined by the model.

The results obtained above are useful to the extent that they can be utilized to achieve an ultimate goal of optionality in design. In other words, the above procedure can be used to generate alternative strategies with associated costs and consequences. But, they provide no information as to how to evaluate these strategies, nor do they have a general framework of choosing an alternative which fits best over our needs and desires. In the next chapter, the general guidelines for this framework are developed, with a view to achieving optional operational strategies for a particular design configuration. Such optionality is discussed from the standpoint of a subjectivistic demand requirements imposed by the users of the facility as the yardstick for comparison of alternative policies.

*When, where, and how much maintenance is applied constitutes a single strategy at an associated cost.

II.5 Summary

In this chapter, the structure of an analytical model for pavement performance has been presented. The model is systems-oriented with causal structure. It consists of three principle components: a structural model, a serviceability-maintenance model, and a cost model.

The structural model is aimed at analyzing the response of the system in an operational environment. The components of the model are so characterized to realistically account for the randomness in the properties of the system and the surrounding environment. Probabilistic estimates of description damage indicators are provided by the structural model. The inputs to this model are categorized into two levels: controllable and non-controllable. The controllable inputs are the geometrical configuration of the system, and the materials properties and their degree of statistical scatter. This scatter defines the level of control on the quality of fabrication and placing of materials to be experienced. The non-controllable inputs are those related to traffic information and environment.

The serviceability-maintenance model is decomposed into two interrelated subsets: serviceability-reliability (S-R) model, and maintenance model. The S-R model predicts the serviceability of the system with associated reliabilities and transition probabilities at any time period. It also determines the life expectancy and distribution of life of the system. On the other hand, the maintenance model is concerned with providing periodical improvement of the state

of the system by introducing maintenance at different time points, and provide a feedback to study the influence of maintenance on the serviceability, reliability and life of the system. Inputs to the S-R model are the damage indicators obtained from the structural model. While inputs to the maintenance model constitute a set of maintenance strategies applied at different points in time which update the state of the system by providing a transition into a state of a higher level of serviceability.

The cost model is not a part of this study, but has been discussed in this chapter because of its relevance to this model and its potential integration with the model to form an overall systems design methodology. This model consists of three components: construction cost model, maintenance cost model, and vehicle operating cost model. The inputs to this model are generated by the structural model and the serviceability maintenance model. The model then yields, for a set of design configuration and maintenance policies, money costs associated with this information.

Alternative design configuration and maintenance policies may be generated by these models and their associated costs are determined to provide information for the final step in the design. This step involves the selection of an optimal design configuration with optimal maintenance policy among a set of alternatives. This choice of an optimal system may be restricted due to budget limitations, availability of material and equipment, competent supervision and control, and so forth. These may be incorporated in an optimiza-

tion model to be developed based on the concepts of linear programming or dynamic programming and decision theories.

CHAPTER III

A DECISION STRUCTURE FOR MAINTENANCE POLICY

III.1. Scope

This chapter is concerned with the development of a decision framework for the choice of alternate maintenance strategies for highway systems. Each strategy defines a set of rules for experimentation and action to which consequences are defined in terms of a multiple set of attributes (D2). In this context, maintenance strategies are generated and evaluated in terms of such attributes. These attributes generally constitute some measures of effectiveness for system's evaluation which may be subjective or objective in nature.

The development of this analysis is based on the Bayesian approach to the theory of decision-making (R1), where a general solution methodology is discussed within this framework utilizing a multi-attribute utility analysis. This technique is then implemented by scaling of the different attributes into monetary values for the utility assessments of various strategies using dynamic programming.

Strategies which yield maximum utilities or minimum costs are the optimal strategies to be selected for maintenance of the system under consideration.

III.2. Framework of the Maintenance Decision Model

The proposed model attempts to incorporate maintenance activities, both in time and in space, in an overall design model, and to develop a framework for the choice of optimal maintenance strategies throughout the design life of the system. The choice of an optimal set of strategies is based on some measures of effectiveness for highway systems as a part of the larger transportation network. The selection of any optimal strategy is effective to the extent that these measures of effectiveness are relevant and exhaustive.

In general, it is possible to derive the measures of effectiveness for highway systems from the function of the overall transportation system. For example, one may state that the purpose of a transportation system is to provide a swift and convenient transportation for the individual and his commerce safely, comfortably, and economically over a range of time. Therefore one can choose factors such as cost, safety, comfort, performance, life, etc. as attributes to measure the adequacy of the highway system to meet its functional requirements, and as a basis to compare alternative systems. Many of these attributes are largely subjective since they encompass the subjective judgements of the users as to what constitutes a satisfactory performance, a safe or a comfortable system. Therefore, the ensuing analysis should be based on expected utilities, which are

expressions of judgemental preferences of individuals for a certain commodity (F3, P3, R1). A tradeoff is then conducted between the different attributes for the selection of the optimal set of strategies.

III.2.1. Description of the Model

The design model described in Chapter II provides information about the Markovian time flows of the states of the system, and generate state and transition probabilities at different time points. The states of the system are described in terms of the serviceability index, which in fact is a utility value for the structural integrity of the system.

Because of the lack of other measures, the AASHO serviceability index is taken to define the performance level of the system at any particular time period in this study. Moreover, one can arbitrarily define the attributes safety, comfort, and cost in addition to serviceability as decision criteria for this problem.

The maintenance performed at any time raises the level of serviceability. This implies a transition to a higher state, how much higher depends on the level of maintenance. Safety and comfort may also be increased due to the upgrading of the system such as increase in skid resistance, filling of potholes, etc. A study along these lines was initiated (L1) and can be pursued to determine the elements of causality and tradeoffs between such factors. In any case, mainte-

nance level is taken to define the effort (in terms of labor and materials) which is expended to upgrade the state of the system from its present level. An associated cost can be assigned for any maintenance effort (A3) and these cost are discounted to their net present worth as a criterion for alternative evaluation. To illustrate this method of approach, let us examine Figure (7) which shows a decision flow diagram depicting the different chance nodes and decision nodes and forks for a typical maintenance decision problem. At the outset, say at time t_1 , the system may be at some state, say s_1 , corresponding to some level of performance with probability p_1 , as determined by the analytical model presented earlier. At this point a decision is encountered: to maintain or not to maintain? And if the decision is to maintain (branch M in the figure), what level of maintenance should be exercised? At this point one can generate numerous branches each of which defines some level of maintenance at an associated cost. However, these branches have been reduced into three highly aggregated levels: a high level (branch h), a medium level (branch m), and a low level (branch l). These three levels can be related to some quantifiable attributes (A3) such as cost of labor, materials and equipment, traffic delays, and so forth. It is assumed that the above maintenance levels have respectively C_1 , C_2 , and C_3 as maintenance cost attributes associated with each level. Furthermore, it is assumed

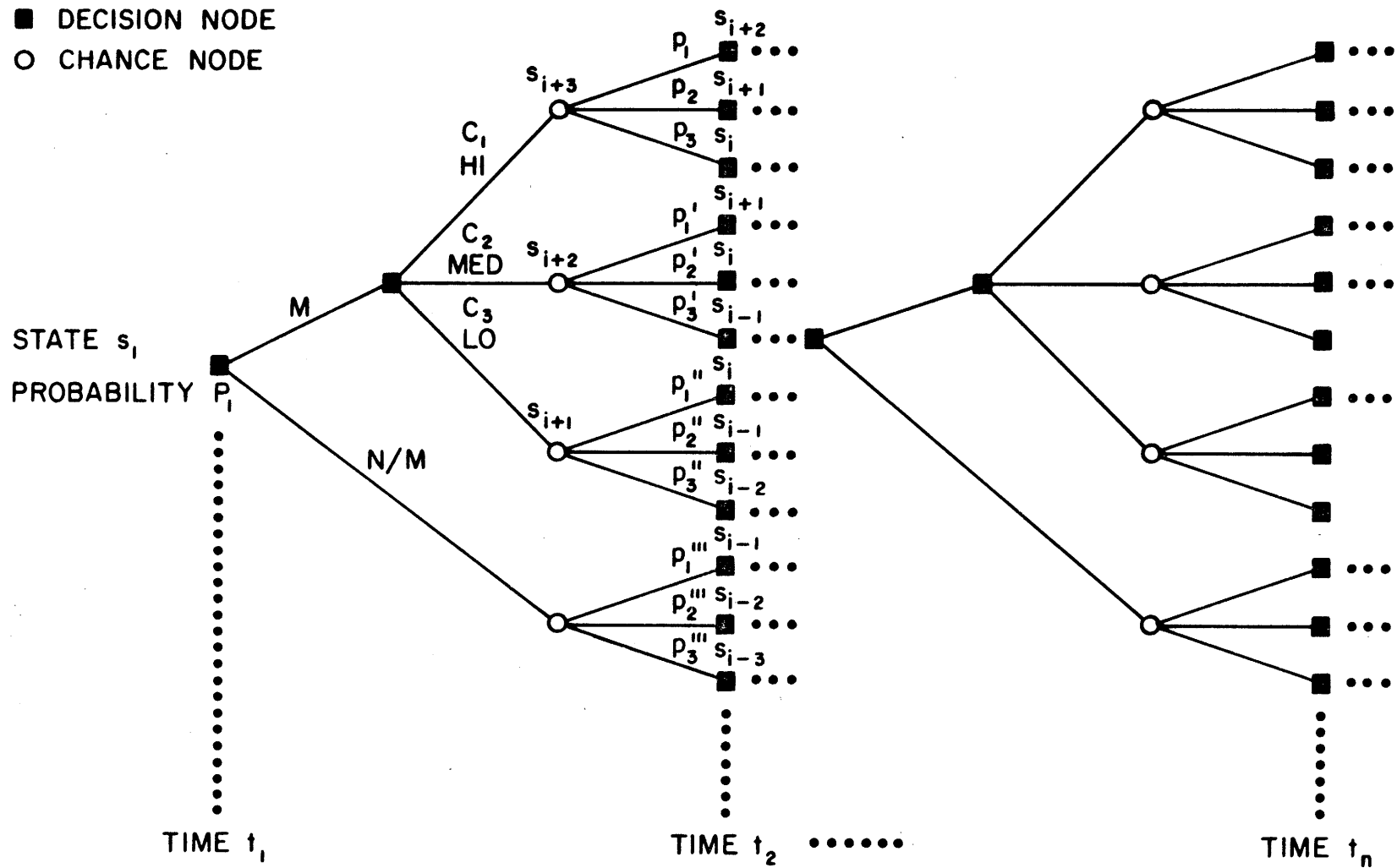


FIGURE 7. DECISION FLOW DIAGRAM FOR MAINTENANCE.

that the choice for the ranges of values for maintenance levels is such that:

- a. There is a definite improvement in the state of the system, once any level has been applied,
- b. this improvement (state transition) takes place almost instantaneously with probability = 1.0, and
- c. the change in the state of the system from its current state i is as follows:

<u>Maintenance Level</u>	<u>New State</u>
High	$i + 3$
Medium	$i + 2$
Low	$i + 1$

If, however, one is to pursue the no-maintenance (N/M) path, the system continues to deteriorate due to the actions of loads and environment and its state may be predicted by the model after any desired time lapse. In fact the no-maintenance path can be combined with the other maintenance levels, and may be labelled as the "zero-maintenance" path with an associated maintenance cost of zero.

If one chooses some fixed time intervals, say one year, at which a decision is taken about the level of maintenance, and the maintenance cost is determined at that point and discounted to a net present value, it is possible to follow the consequences for each path at each time period until the system is rendered non-operational when a major maintenance

effort (resurfacing) is required. These points constitute the tips of the decision tree at which utilities for cost, performance, safety, etc. may be evaluated. The performance of the system as well as the other attributes are evaluated for each one-year period as the average values over that period, for each node. These values may then be accumulated as utilities at the tips of the tree. From that point the procedure of averaging-out-and-folding-back of these utilities is conducted to obtain the maximum expected utility which is normally associated with the optimal strategy for maintenance. The optimal strategy in this case consists of an optimal set of decisions at each of the decision nodes describing the course of action depending on the outcome of the branches preceding the nodes.

The next section deals with the utility analysis for this problem as an approach to decision making.

III.3. Method of Approach

In this section, a general method of approach is formulated using the multiattribute utility analysis to encompass a wide set of attributes and their trade-offs. Later in this section this method is implemented by alternatively scaling down attributes to some monetary equivalent and an example solution is presented for this analysis.

III.3.1. Multiattribute Utility Analysis

Assuming that the decision attributes are: performance,

cost, safety, and comfort, not necessarily in that order of preferences , and that utility functions and indifference curves are available for this analysis, one can use methods as the mid-value splitting technique (Pl, S3) to scale the individual's preference or utilities to each attribute. For instance one can obtain a user's utility for the level of performance of a highway if one can provide an articulated description of the state of the highway. A typical evaluation can be presented in the form of a conversation between an engineer and a prospective user (representing a cross section of users). Remember now that the user is educated and is fully aware of the consequences of his needs and responses: assuming that the scale of performance is between 0.0 to 10.0, worst to best in that order:

Engineer: If you were given the choice between a lottery which yields a 50-50 chance of giving a highway having zero performance or 10.0 performance (perfect), and a highway of 3.5 performance, which one would you settle for?

User: Oh.. 3.5 is rather low, I think I will take the lottery (the implication here is that the user will have to use any of the outcomes of the lottery as his only transportation means).

Engineer: Would you still take the lottery if the performance was 4.5?

User: No. In this case, I definitely will take your offer.

Engineer: (Keeps changing these numbers between the 3.5 and 4.5 values until the user becomes indifferent to either choice, in which case he marks this point, say that it was 3.9 at his midline between zero and one scale in Figure 8).

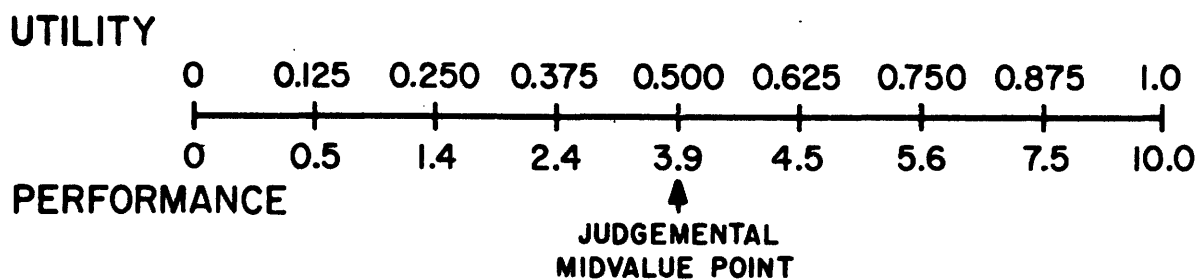


FIGURE 8a. MIDVALUE SPLITTING TECHNIQUE FOR SCALING OF SUBJECTIVE EVALUATION OF PERFORMANCE.

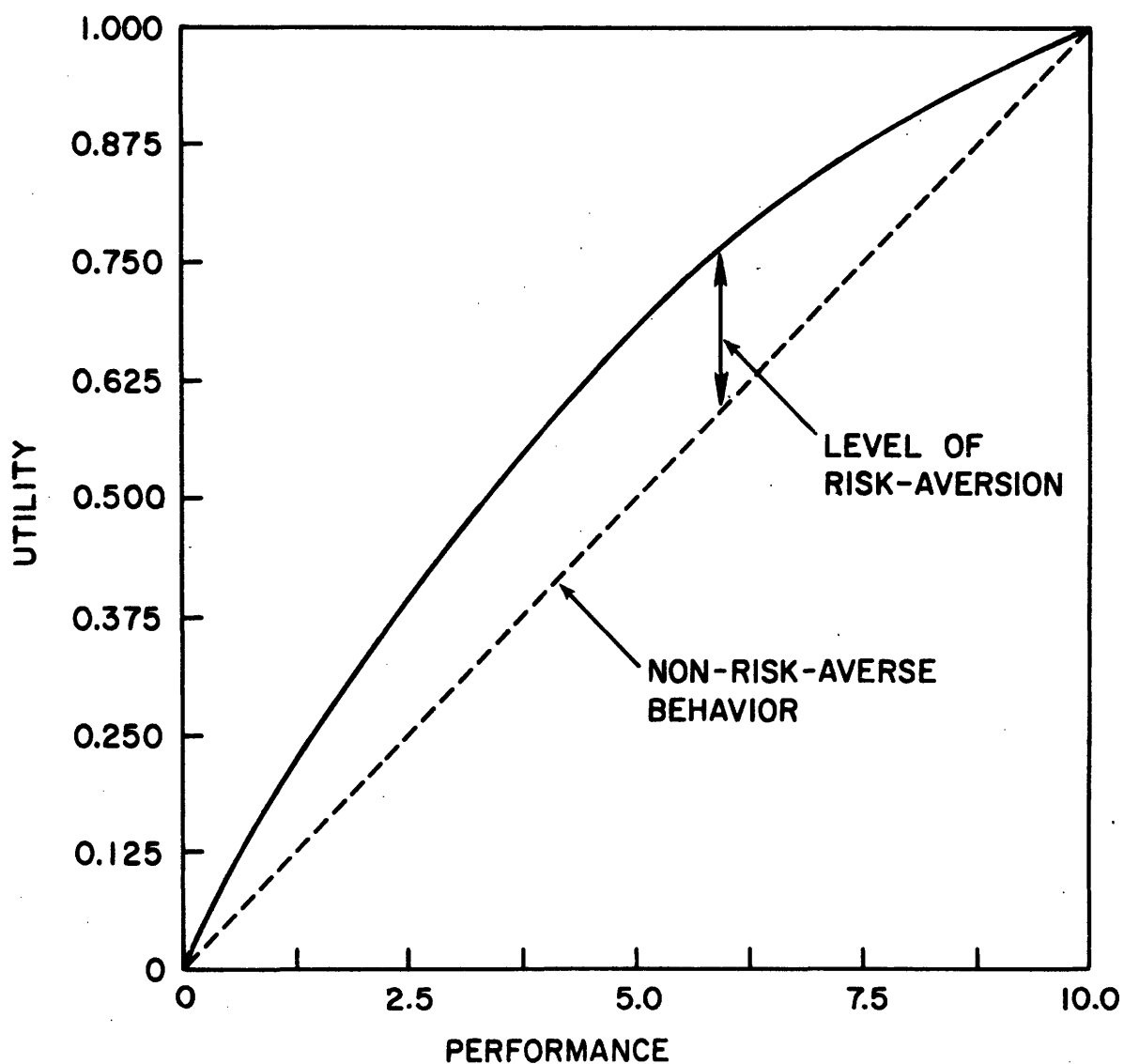


FIGURE 8b. PERFORMANCE UTILITY OR PREFERENCE CURVES OBTAINED BY SCALING.

Now sir, let us see what you would do if you were to choose between a lottery that gives a 50-50 chance 3.9 and 10.0 performance, and a choice of say 5.3?

User: I'll take the lottery.

The engineer continues to raise and lower his values until the user becomes indifferent between them and the lottery. The procedure continues along the same lines, splitting the remainder and finding its lottery equivalent, which is referred to as the mid-value splitting technique (Figure 8a). The final outcome defines the user's utility, or his certainty equivalent to the uncertain. This is shown in Figure 8b. Note that this procedure is easier when dealing with money and other tangible things.

The same procedure can be followed to get the user's attitudes to the three other attributes. However, one can foresee the difficulties that are encountered to get these utilities. Therefore, one can obtain in somehow simpler ways isopreference or indifference curves for the three other attributes with respect to the performance. From this a multi-dimensional surface results. One would hope that these isopreference curves are linear, to reduce the effort of the analysis.

Let us assume, however, that these curves are linear or approximately linear and proceed with the treatment, keeping in mind that a similar procedure is applicable to the nonlinear case with a few adjustments. Consider the

tradeoffs between the performance P and the maintenance cost C . There exists a substitution rate q such that the decrement of one unit of p is equivalent to an increment of q units of C . Figure (9a) shows these tradeoff relationships through the isopreference lines which are similar to typical demand curves. In this figure it is shown that as the performance decreases, the cost of maintenance needed to keep up the system in a "serviceable" condition increases at any fixed time. It is easy to extend this two-dimensional argument into a multi-dimensional space encompassing all other attributes to determine the substitution (tradeoff) rates q_1, q_2 , and q_3 so that the point (p, s, f, c) is indifferent to $(p+q_1 \cdot s+q_2 \cdot f+q_3 \cdot c)$, where p, s, f and c refer to performance, safety, comfort, and maintenance cost, respectively. These multi-dimensional tradeoffs are depicted in Figure (9b).

The evaluation of strategies is best illustrated in Figure (10), which is a representation of the chance nodes described by Figure (7). Each chance node is represented by a lottery with probabilities and consequences shown at the tip of each branch. All multi-dimensional utilities are reduced to scalar utilities expressed in terms of performance indices.

The remainder of the analysis is a straightforward bookkeeping-like process involving averaging-out-and-folding back of those utility numbers obtained above. Each node is evaluated in terms of these utilities, and non-contender

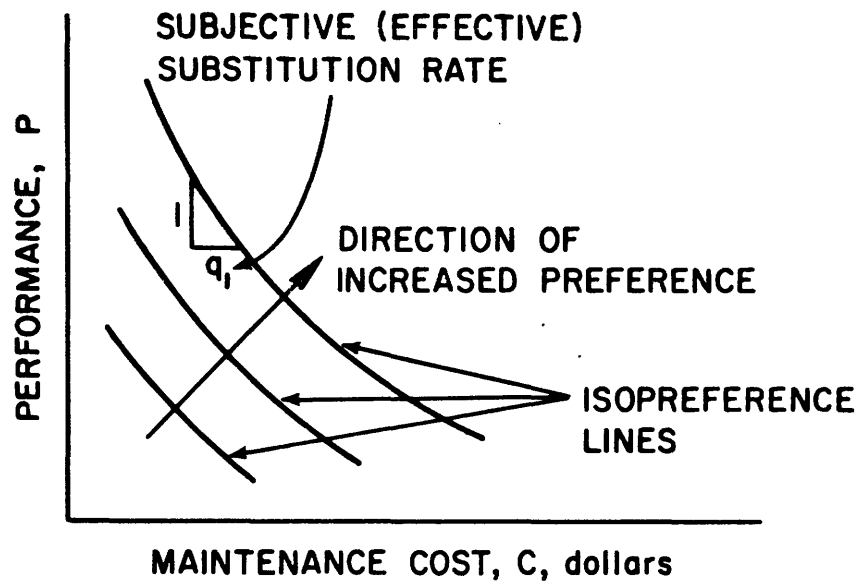
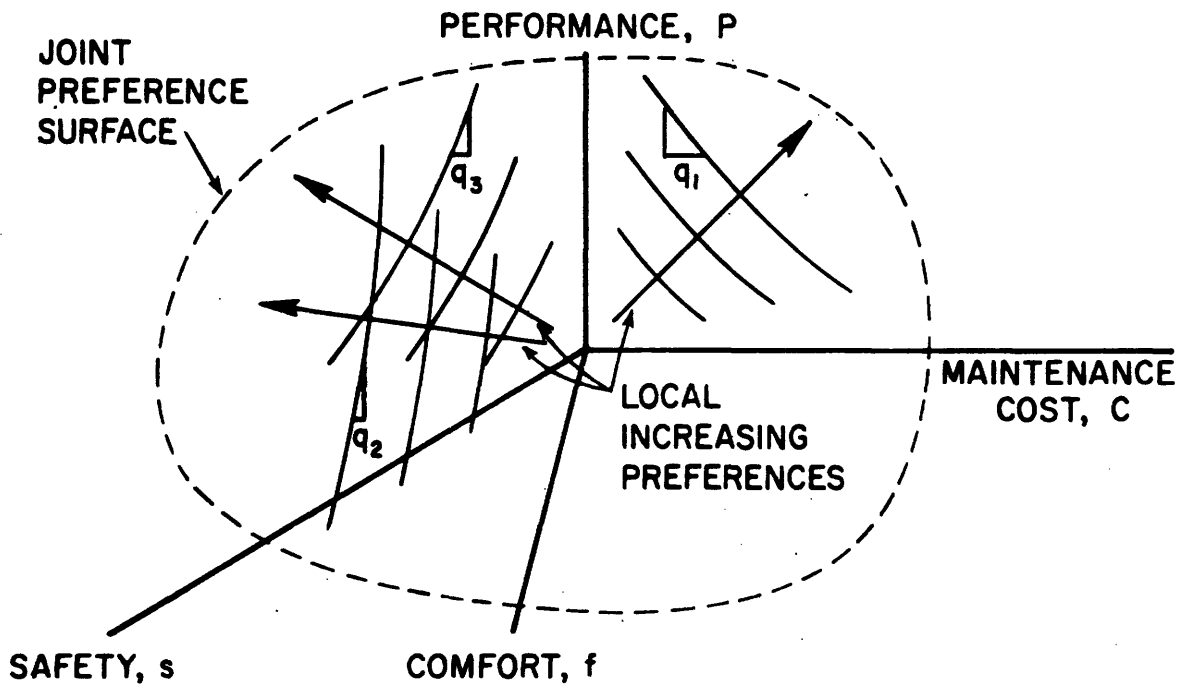
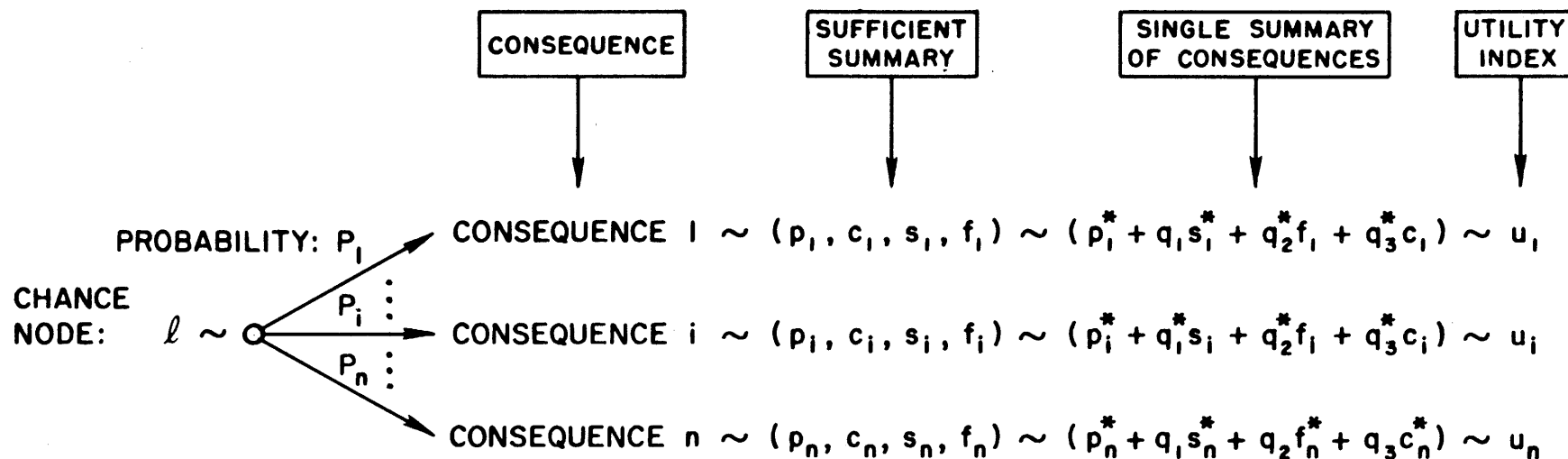


FIGURE 9a. TWO-DIMENSIONAL LINEAR INDIFFERENCE CURVES FOR TRADEOFFS BETWEEN MAINTENANCE COST AND PERFORMANCE.



NOTE: q_1 , q_2 , AND q_3 ARE THE SUBJECTIVE (EFFECTIVE) SUBSTITUTION RATES FOR THESE ATTRIBUTES.

FIGURE 9b. FOUR-DIMENSIONAL INDIFFERENCE CURVES FOR ATTRIBUTE TRADEOFFS OF THE MULTIATTRIBUTE PROBLEM.



$x \sim y$: INDICATES THE PREFERENTIAL INDIFFERENCE BETWEEN ATTRIBUTES x AND y .

$$p^* = p + q_3(c - c^*) + q_1(s - s^*) + q_2(f - f^*).$$

p^*, c^*, s^*, f^* ARE MINIMUM VALUES FOR PERFORMANCE, COST, SAFETY, AND COMFORT REQUIRED FOR THE SYSTEM TO FUNCTION.

FIGURE 10. UTILITY EVALUATION FOR THE MULTIATTRIBUTE PROBLEM.

strategies are eliminated as we proceed backwards through the tree.

Naturally, it is implicitly assumed here that the user or users whose utilities are evaluated are the ones who are paying for the system. In almost all instances there is nobody who is directly paying for the construction or maintenance of any transportation system. The tax money follows a very intricate route before being allocated for these purposes, and therefore it is hard to identify whose preferences must be evaluated. However, one may attempt to circumvent this problem by avoiding a direct confrontation with the problem of who is paying and who is receiving the benefits. Further research in this area is necessary and its solution may not be readily available. One suggested approach may address itself to evaluating existing preferences of highway users by drawing out their preferences of toll roads against ordinary roads. In these cases, the user of the highway is physically paying for a part of his usage for the facility. This is why a simplified and presently-more realistic approach is taken in the implementation phase of this study by scaling utilities into monetary values and dealing with costs as the only attributes in decision-making.

III.3.2. Implementation of the Decision Analysis

More research is needed in the area of multiattribute utility analysis, meanwhile it may be appropriate to adopt

a similar approach but a more tractable one for the decision-making process. This may consist of scaling all attributes to their monetary equivalents, and dealing with those equivalents as imaginary costs or benefits. This approach fits within the framework of the highway cost model established earlier. Alexander and Moavenzadeh (A3) have developed the "Vehicle Operating Cost" which is related to the performance characteristics of the road, its geometry, and factors related the type of the vehicles operating on the road. One can extend this approach further to include "an imaginary cost and/or benefit scale of the users' utility or satisfaction for the performance of the road".

Performance in this case will be taken to encompass safety, comfort, etc. as well as the structural integrity and reliability of the road. Using the highway cost model, one can, at any decision node, assess the total cost incurred from a certain decision regarding the level of maintenance of the highway over a year period by determining its performance over the year and the associated user's cost plus the cost of maintenance involved. All those costs at different time periods are discounted to their net present value to establish a standard or normalized basis of comparison, since costs incurred this year will have completely different value some 10 years later due to inflation, interest of the money, and so forth.

Decisions regarding maintenance strategies are encountered every certain period of time, say every year, for a few years over the life to the system. This constitutes a multi-stage decision problem of the Markov type, since the behavior of the system is Markovian. At each stage, some point in time, a set of decision problems is taken regarding the level of maintenance to be exercised, to which the associated costs C_m are determined. The performance of the system over the whole year is integrated to determine vehicle operational costs for each decision in the following manner:

$$C_v = \int_{t_1}^{t_2} P(\tau) \dot{C}_v(\tau) d\tau \quad (3.1)$$

Where:

C_v = the vehicle operational costs for the period
($t_2 - t_1$)

$p(\tau)$ = the performance at time

$c(\tau)$ = the corresponding cost incurred from $p(\tau)$

The two cost components are discounted to their net present worth and added together to form a single cost factor widely known as a reward (H_1, N_1) which is the consequence of each of the decisions taken at that stage measured over a one-year period.

The optimization mechanism for maintenance strategies utilizes the dynamic programming approach for a multi-stage process. This is done by introducing the decision-making process at each stage, permitting a choice among several transition and cost (or return) matrices. A decision variable $d_n=k$, for $k = 1, \dots, K$ designates the choice of the k th transition matrix and the k th associated cost or return matrix at the n th stage. More specifically, if the system is at state i , $d_n=k$ means that the relevant transition probabilities and costs at stage n are the i th row of the k th transition matrix and the i th row of the k th cost matrix. The transition matrix and the cost matrix are shown by equation (3.2)

$$\underline{P} = \begin{bmatrix} p_{11} & \dots & p_{ij} & \dots & p_{1M} \\ p_{i1} & \dots & p_{ij} & \dots & p_{iM} \\ p_{M1} & \dots & p_{Mj} & \dots & p_{MM} \end{bmatrix} \quad (3.2a)$$

$$\underline{C} = \begin{bmatrix} c_{11} & \dots & c_{ij} & \dots & c_{1M} \\ c_{i1} & \dots & c_{ij} & \dots & c_{iM} \\ c_{M1} & \dots & c_{Mj} & \dots & c_{MM} \end{bmatrix} \quad (3.2b)$$

The transition matrix P contains the transition probabilities from state i to state j for $i, j = 1, \dots, M$, while the cost matrix C gives the cost associated with such a transition over a certain period of time. This basically include the maintenance cost plus the vehicle operator's cost over this period. The total expected cost for an n -stage Markov process starting in stage i , expressed recursively, is the expected cost in stage n plus the expected cost in stage $(n-1)$ from the resulting state, summed over all states. For a one-stage process,

$$\bar{C}_1(i) = \sum_{j=1}^M P_{ij} C_{ij} \quad (3.3)$$

and for an n stage process

$$\bar{C}_n(i) = \sum_{j=1}^M P_{ij} [C_{ij} + \bar{C}_{n-1}(j)] \quad (3.4)$$

Generally, there is no need to assume that the same matrices are available for selection at each stage, and that the transition and cost matrices are chosen in pairs rather than individually (N1). Then assumptions are made here only for the simplification of the analysis.

Define the probability transition by $P_{ij}(d_n)$, and the cost by $C_{ij}(d_n)$. The expected cost for n stages, starting in state i is computed recursively to be:

$$\bar{C}_1(d_n/i) = \sum_{j=1}^M P_{ij}(d_1) C_{ij}(d_1) \quad (3.5)$$

and

$$\begin{aligned} \bar{C}_n(d_n, d_{n-1}, \dots, 1/i) = & \sum_{j=1}^M P_{ij}(d_n) [C_{ij}(d_n) + \\ & \bar{C}_{n-1}(d_{n-1}, d_{n-2}, \dots, 1/j)], \quad n=2, \dots, N \end{aligned} \quad (3.6)$$

To minimize the expected cost from N stages as a function of the initial state, let:

$$\begin{aligned} \phi_N(i) = & \text{Min.}_{d_N, \dots, d_1} \sum_{j=1}^M P_{ij}(d_N) [C_{ij}(d_N) + \bar{C}_{N-1} \\ & [d_{N-1}, d_{N-2}, \dots, d_1 | j]] \end{aligned} \quad (3.7)$$

Applying recursive optimization, we get

$$\phi_1(i) = \min_{d_1=1, \dots, k} \sum_{j=1}^M p_{ij}(d_1) c_{ij}(d_1) \quad (3.8)$$

and

$$\phi_n(i) = \min_{d_n=1, \dots, k} \sum_{j=1}^M p_{ij}(d_n) [c_{ij}(d_n) + \phi_{n-1}(j)]$$

for $n = 2, 3, \dots, N$ (3.9)

In equation (3.8) and (3.9), the term $\sum_{j=1}^M p_{ij}(d_n) c_{ij}(d_n)$ denotes the expected cost in stage n , while the second term includes expected cost for all stages before n .

If we let

$$q_i(d_n) = \sum_{j=1}^M p_{ij}(d_n) c_{ij}(d_n) \quad (3.10)$$

The recursive equations (3.8) and (3.9) can be expressed in a simpler notational form as follows:

$$\bar{\phi}_1(i) = \min_{d_1=1, \dots, k} q_i(d_1), \quad i = 1, \dots, M \quad (3.11)$$

$$\phi(i) = \min_{d_n=1, \dots, k} [q_i(d_n) + \sum_{j=1}^n p_{ij}(d_n) \phi_{n-1}(j)],$$

$d_n = 1, \dots, k$ (3.12)

for $i = 1, \dots, M$ and $n = 2, \dots, N$

The above recursive solution is sometimes referred to as value iteration solution (H2). It forms a special case

(Markovian) of the stochastic multi-stage optimization, and can be solved by the usual dynamic programming computational techniques.

III.3.3. Numerical Example of Maintenance Optimization by Dynamic Programming

To illustrate the solution of a Markovian decision process for optimal maintenance strategies utilizing dynamic programming techniques, a simple hypothetical example is presented below:

This example involves four stages of decision-making spaced at one year apart. There are two possible decisions at each stage each involves a certain level of maintenance:

- a. Light Maintenance: $d_n=1$, with transitions and costs as follows:

$$\bar{P}_1 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}, \quad \bar{C}_1^* = \begin{bmatrix} 0 & 6 \\ 4 & 8 \end{bmatrix}$$

- b. Medium Maintenance: $d_n = 2$, with

$$\bar{P}_2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad \bar{C}_2 = \begin{bmatrix} 2 & 4 \\ 1 & 0 \end{bmatrix}$$

* The \bar{C}_1 matrix includes yearly costs associated with state transitions, for example C_{11} in this matrix equals zero which implies that no maintenance costs or users cost are accrued in this transition.

Table (3.1) - First Year Decisions

i	d_1	$q_i(d_1)$	$\bar{\phi}_1(i)$	$d_1(i)$
1	1	$1/2(0)+1/2(6)=3.$	2.	(2)
	2	$1(2)+0(4)=2.$		
2	1	$3/4(4)+1/4(8)=7.$	0.5	(2)
	2	$1/2(1)+1/2(0)=0.5$		

Table (3.2) - Second Year Decisions

i	d_2	$q_i(d_2) + \sum_{j=1}^2 p_{ij}(d_2)\bar{\phi}_1(j)$	$\bar{\phi}_2(i)$	$d_2(i)$
1	1	$3+1/2(2)+1/2(.5)=4.25$	4.	(2)
	2	$2+1(2)+0(.5)=4.$		
2	1	$7+3/4(2)+1/4(.5)=8.63$	1.75	(2)
	2	$.5+1/2(2)+1/2(.5)=1.75$		

Table (3.3) - Third Year Decisions

i	d_3	$q_i(d_3) + \sum_{j=1}^3 p_{ij}(d_3)\bar{\phi}_2(j)$	$\bar{\phi}_3(i)$	$d_3(i)$
1	1	$3+1/2(4)+1/2(1.75)=5.88$	5.88	(1)
	2	$2+1(4)+0(1.75)=6.$		
2	1	$7+3/4(4)+1/4(1.75)=10.43$	3.38	(2)
	2	$.5+1/2(4)+1/2(1.75)=3.38$		

Table (3.4) - Fourth Year Decisions

i	d_4	$q_i(d_4) + \sum_{j=1}^4 p_{ij}(d_4)\bar{\phi}_3(j)$	$\bar{\phi}_4(i)$	$d_4(i)$
1	1	$3 + 1/2(5.88) + 1/2(3.38)$	7.63	1
	2	$2 + 1(5.88) + 0(3.38)$	7.88	
2	1	$7 + 3/4(5.88) + 1/4(3.38)$	5.25	2
	2	$0.5 + 1/2(5.88) + 1/2(3.38)$	5.13	

The computations and solutions are tabulated in Tables 3.1, 3.2, 3.3, and 3.4 below, for four stages.

For this example the results indicate that at stages one and two one has to use strategies number 2, which correspond to $\phi_1(1) = 2.0$, $\phi_1(2) = 0.5$ in the first year (Table 3.1), and $\phi_2(1) = 4.0$, $\phi_2(2) = 1.75$ in the second year (Table 3.2). Later, however, there is a tendency to a steady behavior in which one has to use strategy number 1 if the system was initially in state 1, and strategy number 2 if the system was in state 2.

A larger number of decision strategies and larger number of stages requires a computer program to analyze, but it follows the same simple analysis presented above.

It is worthwhile to comment on the results obtained in the above example. It is noticed that the optimal decisions in all stages at each state becomes the same after a certain number of stages, i.e., independent of the stage number. This phenomenon is referred to as the steady state or asymptotic behavior in Markov processes. A steady state behavior is one in which the behavior of the system becomes independent of its initial states after a certain period of time.

The effects of discounting may be introduced through the cost matrices at each stage by multiplying the cost at the particular stage by an appropriate discounting factor, such that all costs are expressed in terms of their net

present values.

The approach presented above and illustrated by the numerical example constitutes a powerful tool for the solution of this class of problems. The development of a computerized model which utilizes the recursion formulation presented in equation (3.11) and (3.12) and illustrated computationally by Tables (3.1) through (3.4) should not constitute a major effort.

IV. MODEL VALIDATION AND SENSITIVITY ANALYSIS

IV.1. Purpose

This chapter is concerned with the testing and validation of the models presented in Chapter II. A limited sensitivity study is also conducted to examine trends in the behavior of pavement systems, and to draw comparisons between the results of this analysis and the expected behavior of the real-world systems. The above models have been coded into a set of computer programs which are briefly described below.

In the ensuing analysis, several systems are examined; they vary in both their geometries and the mechanical properties of the constituent materials. Further, the influence of quality control levels on the behavior of the system is also examined. In this respect several quality control levels are studied in terms of their effects on both the damage progression and the serviceability, reliability, and life of the system.

IV.2. Description of the Computer Programs

The computer programs developed for this study derive their nature and operation from those of the above models. They are coded in FORTRAN IV language, suitable for an IBM System/370 or IBM System/360. They are also equipped with an optional graphical display routine which operates on a

Stromberg-Carlson 4020 System, a cathode ray plotting device, providing graphical outputs on photo prints or micro films.

Table (4.1) shows a typical set of inputs for these programs. The inputs are divided into four categories: system geometry, materials' properties, load characteristics, and temperature history.

- A. System Geometry: This is expressed in terms of the heights of the first and second layers.
- B. Materials' Properties: These are divided into two components: creep or elastic properties, and fatigue characteristics.

1. Creep or Elastic Properties: which are expressed in terms of elastic or viscoelastic compliance functions obtained from laboratory tests for the materials in each layer. Normally, these properties are statistically distributed; laboratory tests provide several creep curves for each type of material. The creep compliances are fitted to an exponential (Dirichlet) series of the form:

$$D(t) = \sum_{i=1}^N G_i \exp(-t\delta_i)$$

using the least squares fitting technique. The resulting coefficients and exponents for each layer, along with the system's geometry are used as inputs to a static load program. The outputs of this program are expressed in terms of time flows of stresses, strains, or deflections at any point within

the system. These represent the response of the system to a step load; which may be referred to as the "system characteristic functions". The above responses are again fitted to an exponential series representation, utilizing the least squares fitting procedure, to yield a set of G_i 's and δ_i 's as discussed above. For this study, the response functions are obtained in terms of the normal deflections at the surface of the system, and the tensile strains at the interface between the first and second layers. The former is used to obtain estimates of the rutting and slope variance, while the latter provides estimates of the area of cracking within the system. The above response functions are expressed in terms of means and variances or coefficients of variations, as shown in Table (4.1), Appendix III.

In addition to the creep functions, temperature shift factors for the creep functions of the system are determined for the particular temperature history at hand. These are shown in the last part of Table (4.1).

2. Fatigue Characteristics: these are the coefficients K_1 and exponents K_2 for the fatigue relationship of the system, which is expressed as:

$$N_f = K_1 \left(\frac{1}{\Delta \epsilon} \right)^{K_2} \quad (4.1)$$

where: N_f represents the number of loads to failure, and

$\Delta \epsilon$ is the tensile strain amplitude.

These coefficients are generally sensitive to temperature and are also statistical in nature. They are determined in this particular study using the following expressions:

$$K_1 = 10 [-6.0 - 0.15(106 - T)] \quad (4.2)$$

$$K_2 = 2.0 + (120 - T)^{0.40} \quad (4.3)$$

where: T is the prevailing temperature in degrees Fahrenheit. Further, the spatial correlation parameters are also inputs to the program. These are the parameters B and C in the following expression:

$$\rho = A + B \exp(-|x^2|/C^2) \quad (4.4)$$

Equation (4.4) is the same as equation (2.26) discussed in Chapter II.

C. Load Characteristics: These include

1. Radius of the applied loads in inches
2. Intensity of loads, in lbs/in²
3. Load amplitude, which is a linear multiplier of the load intensity
4. Duration of the loads in seconds
5. Rate of load applications per month and the proportion of channelized loads.

- D. Temperatures: This is a vector of monthly temperature history for the region at hand.

Typical outputs of these programs are shown in the last part of Table (4.1). These outputs include the following:

- A. Response History: Which is expressed in terms of statistical estimates of AASHO's damage indicators measured at different time periods.
- B. Performance Characteristics: Which include
 - 1. Means and variances of the AASHO's present serviceability index at different time periods.
 - 2. The reliability of the system at various points in time, and
 - 3. The probability mass function, cumulative function, and expected value of the life of the system.

In addition, the program provides estimates of the marginal state probabilities at different time points, and if desired, the probabilities of state transition for the system.

The above outputs can be provided both in tabular and graphical forms. A listing of the computer programs may be found in Appendix II. The next section presents some numerical analyses for various systems conducted as a part of a sensitivity study to illustrate the capabilities of the models to predict behavioral patterns for these systems.

IV.3. Sensitivity Analysis

In this section, several systems are examined with a view to identify the sensitivity of the behavior of these systems to various design parameters. Further, comparisons are drawn among these systems to establish the level of agreements between the predicted patterns of behavior and those anticipated in the real-world systems.

The parameters investigated are essentially those which an engineer can specify when designing a highway. They include: the geometry of the system, the physical properties of the material constituents, and the level of quality control to be exercised on the system fabrication. To establish a common yardstick for comparison among the different systems at hand, these systems were subjected to similar load and temperature histories. These histories are shown in Table (4.1) below, and are repeated for all the cases examined henceforth. All tables for these analyses are presented in Appendix III.

IV.3.1. Influence of the Geometric Factors

This section examines the influence of the system's geometry on its future performance. The system at hand is labelled as System 1, with three different geometries, namely: thin, medium, and thick. The geometry of the system is expressed in terms of the heights of its first and second layers, with the third layer being infinitely deep. Accordingly, the above three geometries are as follows:

<u>Height, in Inches</u>		
<u>Geometry</u>	<u>First Layer</u>	<u>Second Layer</u>
Thin	3	3
Medium	4	6
Thick	5	9

The properties of the materials in each case are kept the same for all the three different geometries. Tables (4.1), (4.2), and (4.3) present the inputs and outputs for the thin, medium, and thick geometries, correspondingly. The relevant responses of the system to a step load are obtained using the static load and the curve fitting programs. These are tabulated under the heading "Mechanical Properties" in each table. The load and temperature characteristics are also listed. Probabilistic estimates of the damage indicators, expressed in terms of rut depth, slope variance, and cracking are obtained under random loading history at different time periods. Further, the serviceability, reliability, and distributions of life are also obtained for each case.

Figures (11), (12), and (13) show the rut depth, slope variance, and cracking histories for the three geometries. These figures show that as the system becomes thicker, the level of damage is reduced. Accordingly, the serviceability level of the system and its reliability increase. This becomes apparent from observing Figure (14) which shows the serviceability, reliability, and distribution of life for the three geometries. From this figure, it is shown that

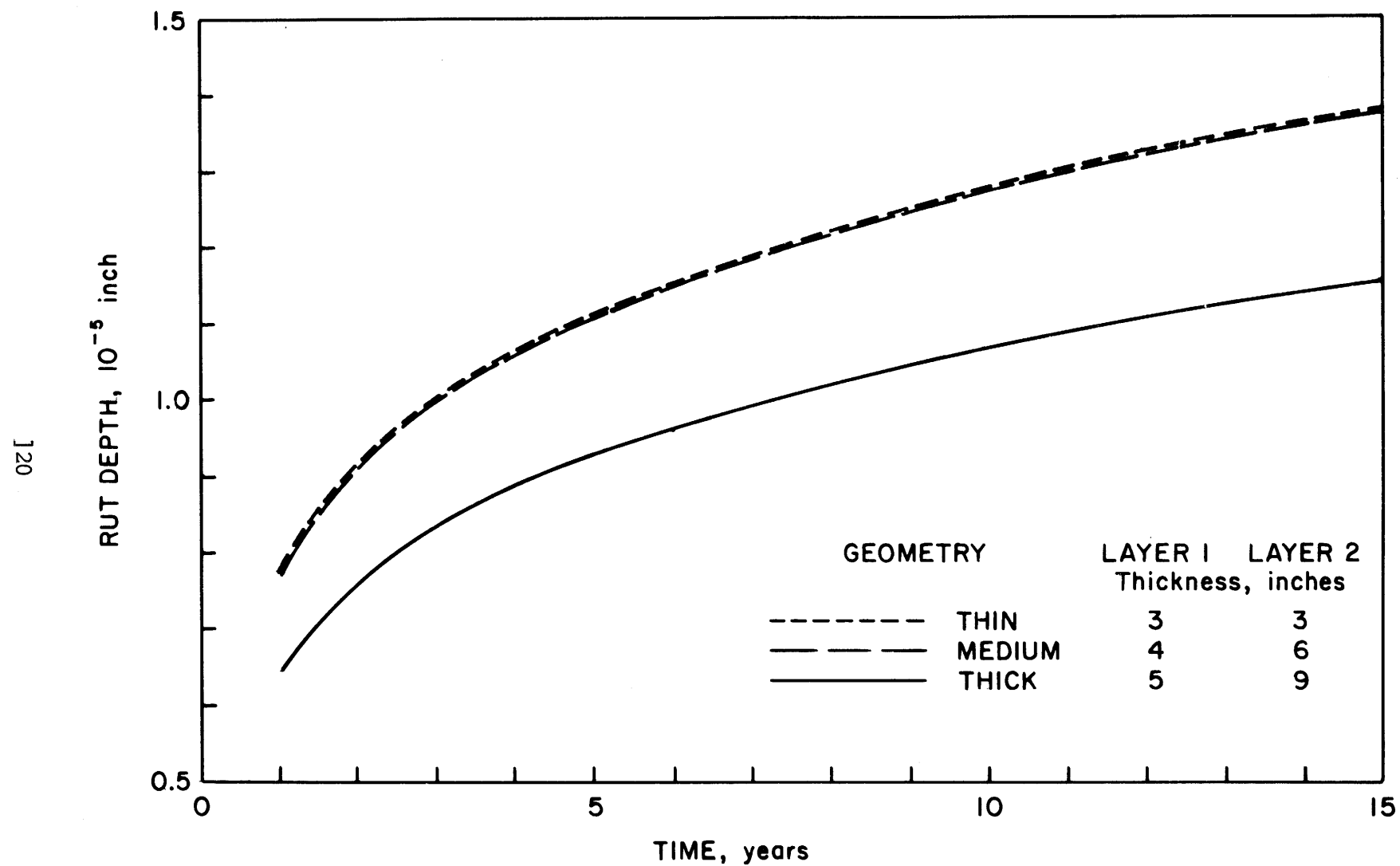


FIGURE 11. INFLUENCE OF GEOMETRY ON RUTTING HISTORY.

the life expectancy of the system increases with the increase in its geometry.

The last parts of Tables (4.1) through (4.3) also present the marginal probabilities of states at different points in time. These are shown in Figure (15), where two observations may be of interest for this study. These are:

1. For each geometry, as time elapses the probability distributions shift to the right in the direction of lower serviceability level. This implies that after three years for example, the probability of being in state 1 is approximately 8 times less than its value after one year only. This probabilistic pattern of observation is in agreement with the intuitive expectation that a system is more likely to be in a lower performance state as time elapses if no maintenance is applied.

2. When comparing figures (15a) and (15c), it is observed that as the system becomes thicker, the probabilities of being in states of higher serviceability are higher at any time point. Therefore, the probabilities of being in states 1 and 2 are higher for the thicker system, while the probabilities of being in state 10, i.e., the failure state, are consistently lower.

While the numerical values provided by these example may be rather hypothetical, the patterns of the above observations provide a reasonable agreement with those

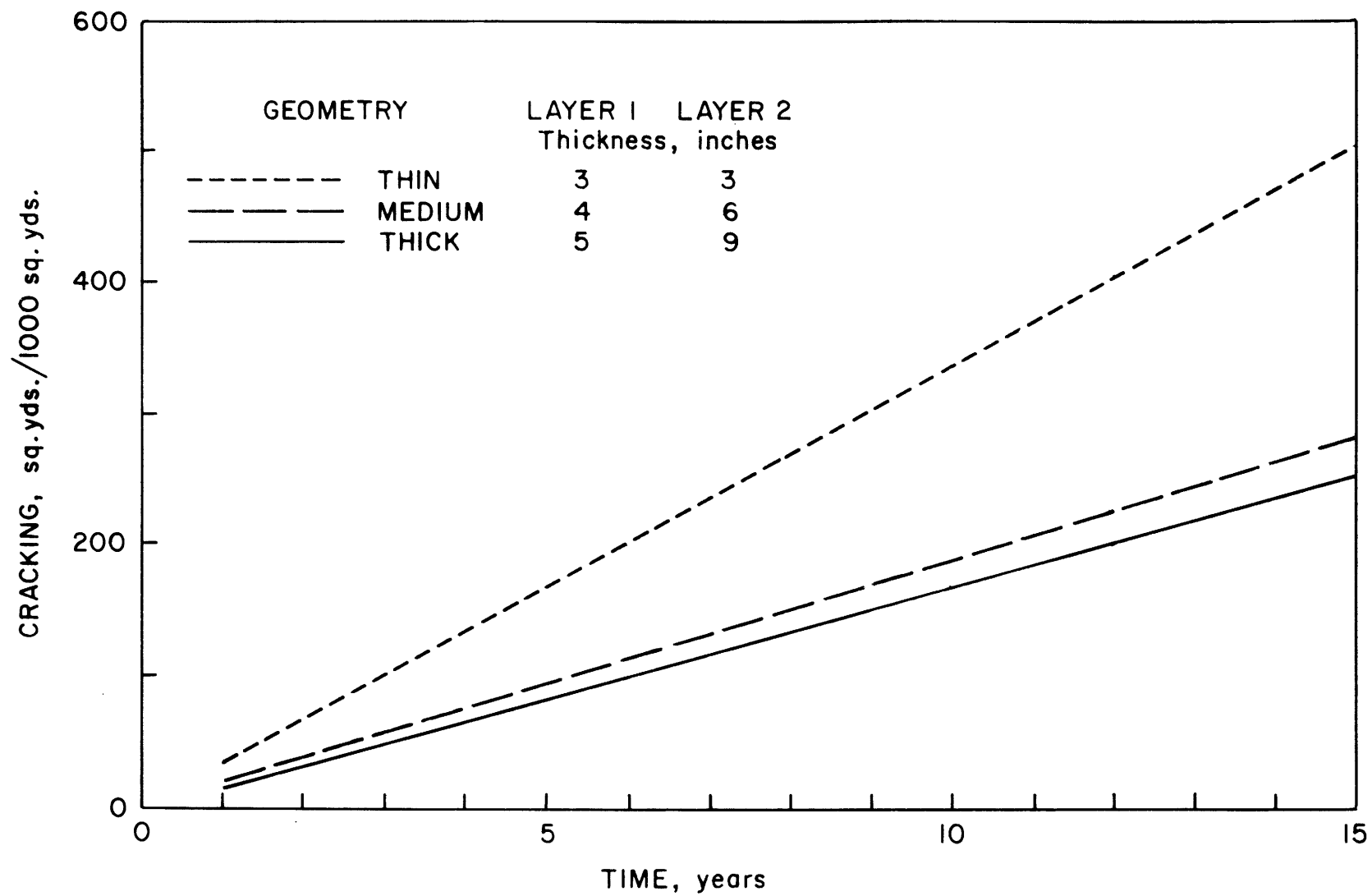
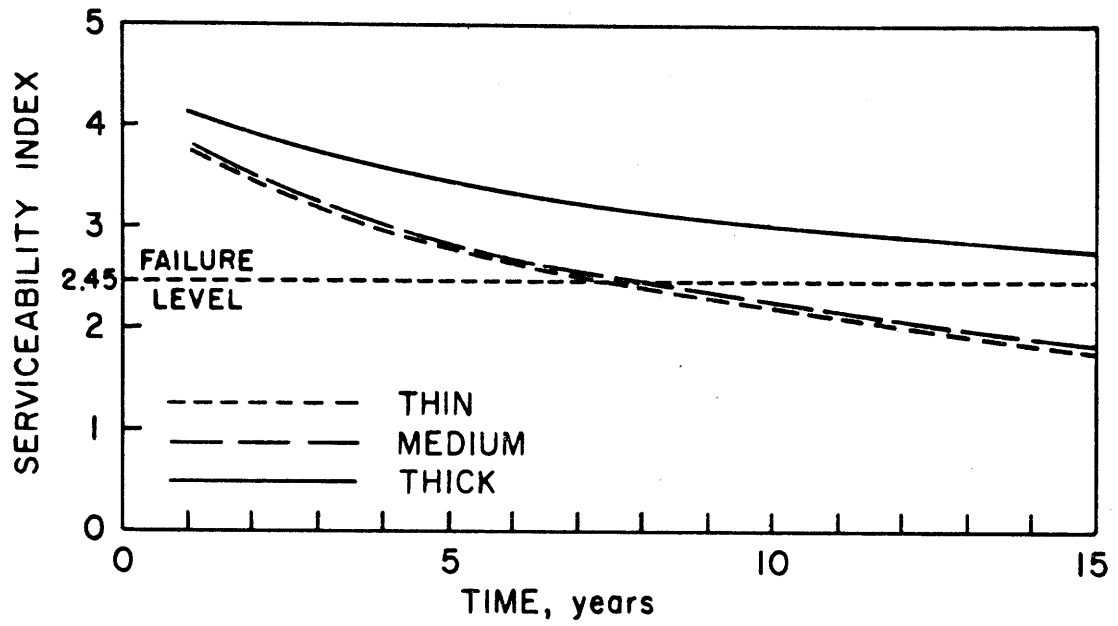
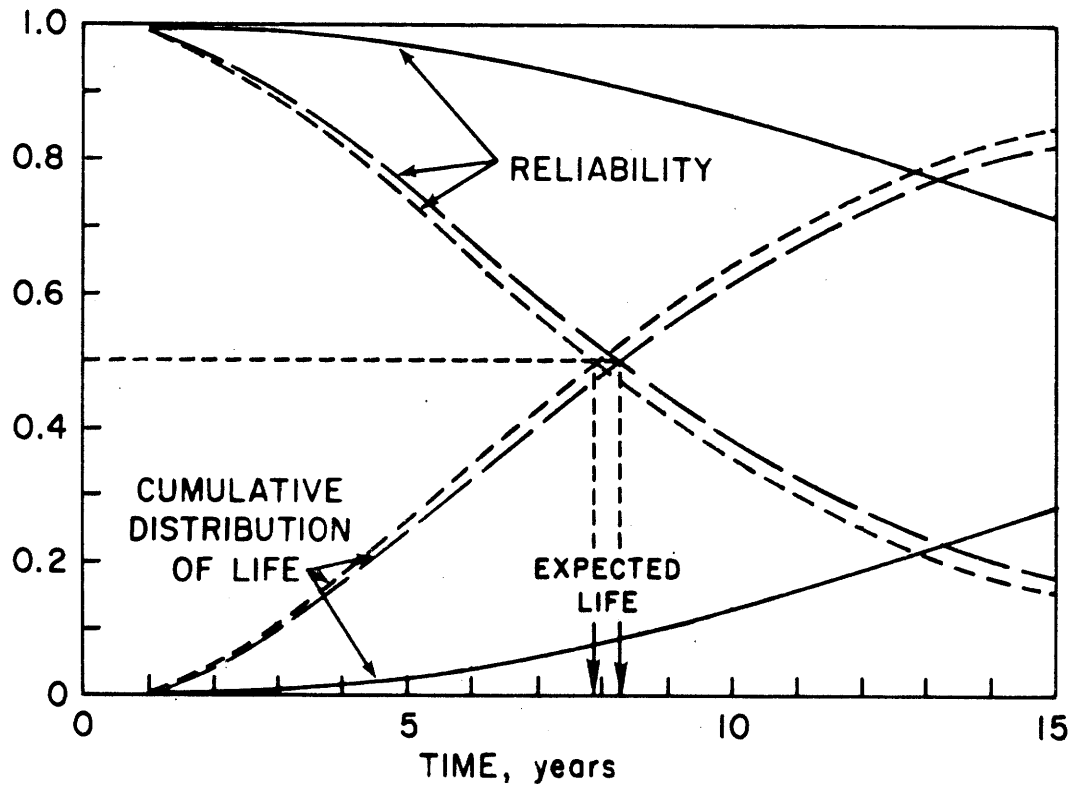


FIGURE 13. INFLUENCE OF GEOMETRY ON CRACK PROGRESSION.



(a)



(b)

FIGURE 14. INFLUENCE OF GEOMETRY ON (a) SERVICEABILITY INDEX, AND (b) RELIABILITY AND LIFE EXPECTANCY.

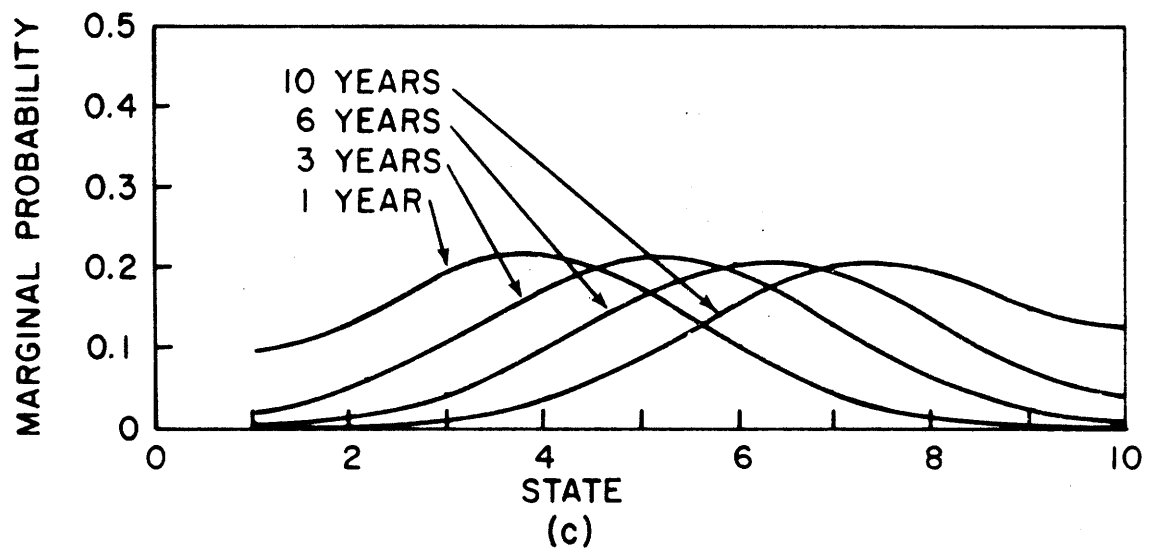
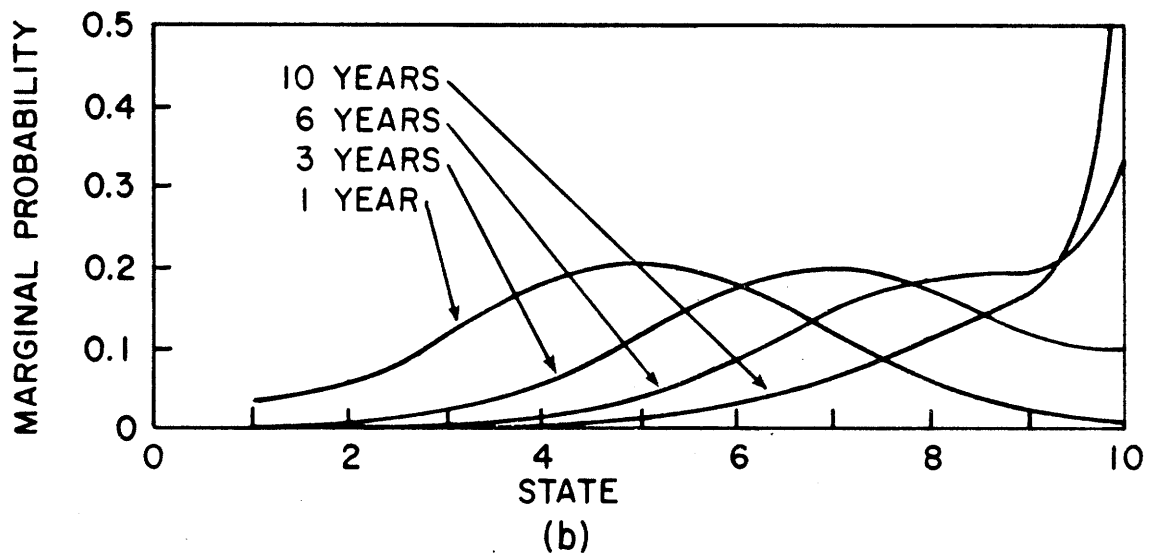
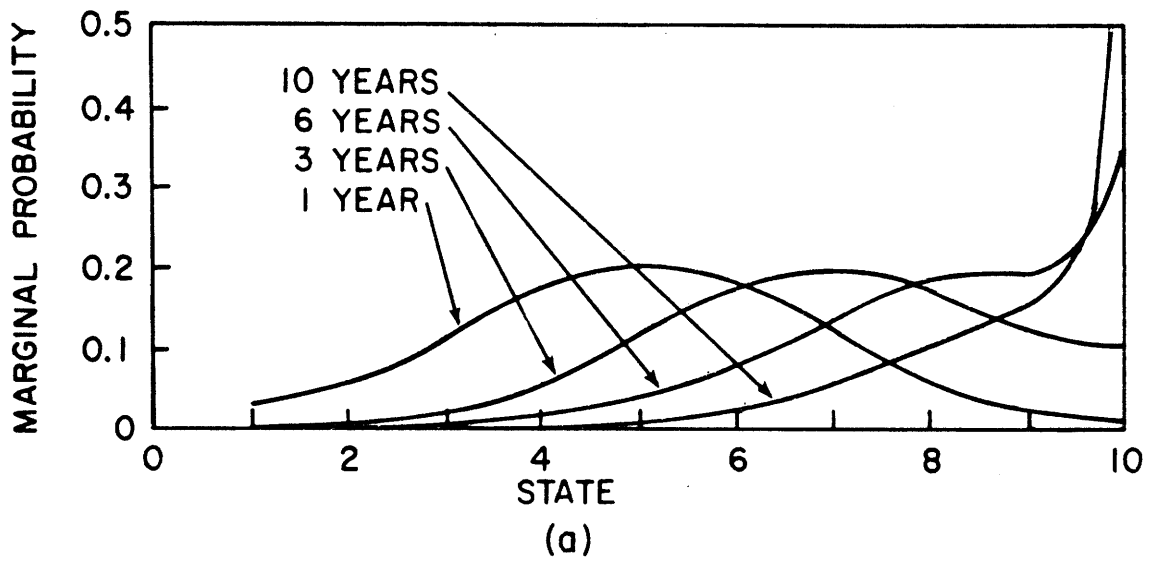


FIGURE 15. MARGINAL PROBABILITIES FOR (a) THIN, (b) MEDIUM, AND (c) THICK, SYSTEMS.

the life expectancy of the system increases with the increase in its geometry.

The last parts of Tables (4.1) through (4.3) also present the marginal probabilities of states at different points in time. These are shown in Figure (15), where two observations may be of interest for this study. These are:

1. For each geometry, as time elapses the probability distributions shift to the right in the direction of lower serviceability level. This implies that after three years for example, the probability of being in state 1 is approximately 8 times less than its value after one year only.

This probabilistic pattern of observation is in agreement with the intuitive expectation that a system is more likely to be in a lower performance state as time elapses if no maintenance is applied.

2. When comparing figures (15a) and (15c), it is observed that as the system becomes thicker, the probabilities of being in states of higher serviceability are higher at any time point. Therefore, the probabilities of being in states 1 and 2 are higher for the thicker system, while the probabilities of being in state 10, i.e., the failure state, are consistently lower.

While the numerical values provided by these example may be rather hypothetical, the patterns of the above observations provide a reasonable agreement with those

expected in the real-world systems. It is generally known from experience that as the thicknesses of the layers in the system increase, the system's performance is enhanced, and consequently, its life expectancy is increased.

IV.3.2. Influence of Materials' Properties

In this section, three systems of various properties are chosen, to study the sensitivity of the model to changes in the properties of the materials in the systems. These systems are categorized as: weak, medium, and strong, depending on the creep functions of their constituent materials. The inputs and outputs for this analysis are presented in Tables (4.1), (4.4), and (4.5), where System 1, System 2, and System 3 correspond to the medium, weak, and strong properties, respectively.

The results of this analysis are presented in Figures (16) through (20). Figures (16), (17) and (18) present a comparison among the histories of rutting, slope variance, and cracking for the three systems. Further, Figure (19) presents a comparison for the serviceability, reliability, and life expectancy among the three systems.

Again, as it may be expected, as the system becomes stronger, the damage formation and progression becomes slower. There was one exception, where the cracking component in the weak system was smaller than that in the medium system. This may be attributed to the fact that although system 2 is weaker than system 1 at long times, its instantaneous response

characteristics are higher than those of system 1. This may be observed from Tables (4.1) and (4.4). In those tables the sum of the radial strain coefficients represent the instantaneous response of the system to external excitations. The sum of the strain coefficients for system 1 is 0.00562 Table (4.1) while the corresponding sum for System 2 is 0.00512 Table (4.4). Since the strain amplitudes causing cracking occur immediately after the load cycle, System 1 is expected to undergo more cracking than System 2, despite its higher strength at longer times.

From Figure (19a), it is observed that the serviceability level increases with the increase in the strength of the system. The reliability and life expectancy of the system also are higher for stronger systems as depicted in Figure (19b). Since the level of damage in System 3 is very low, its serviceability and reliability are maintained at some rather high levels, and its life expectancy falls beyond the time domain of this analysis (which is 15 years). Unrealistic as it may seem, this result provides only a behavioral pattern in which the stronger system possesses a higher resistance to damage initiation and progression, and consequently a higher life expectancy. The accuracy of the numerical values is dependent on the inputs which are obtained from experimental and field observations. As this information becomes gradually available, the model is continuously calibrated and adjusted to fit the relevant

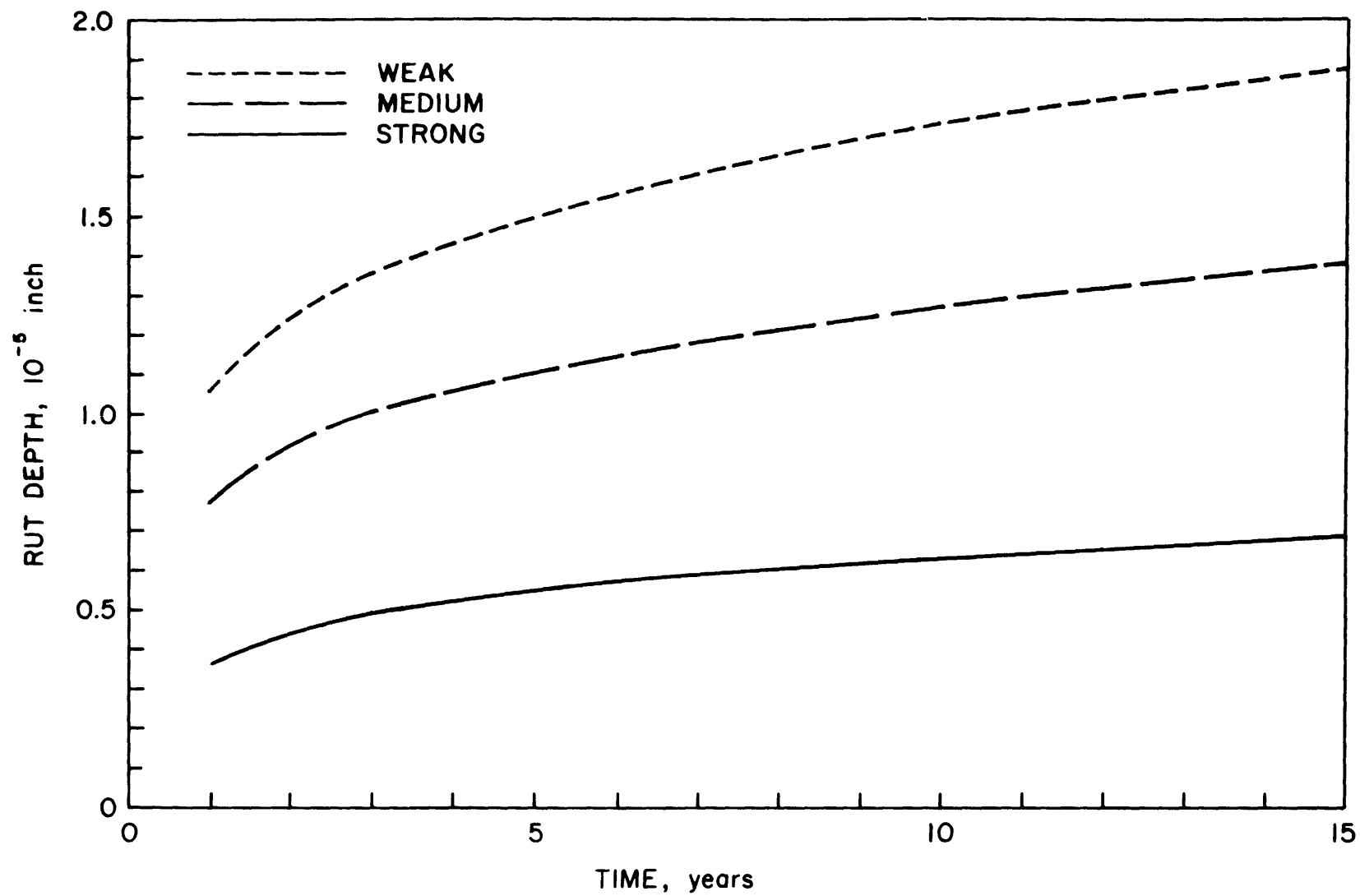


FIGURE 16. INFLUENCE OF MATERIAL PROPERTIES ON RUTTING HISTORY.

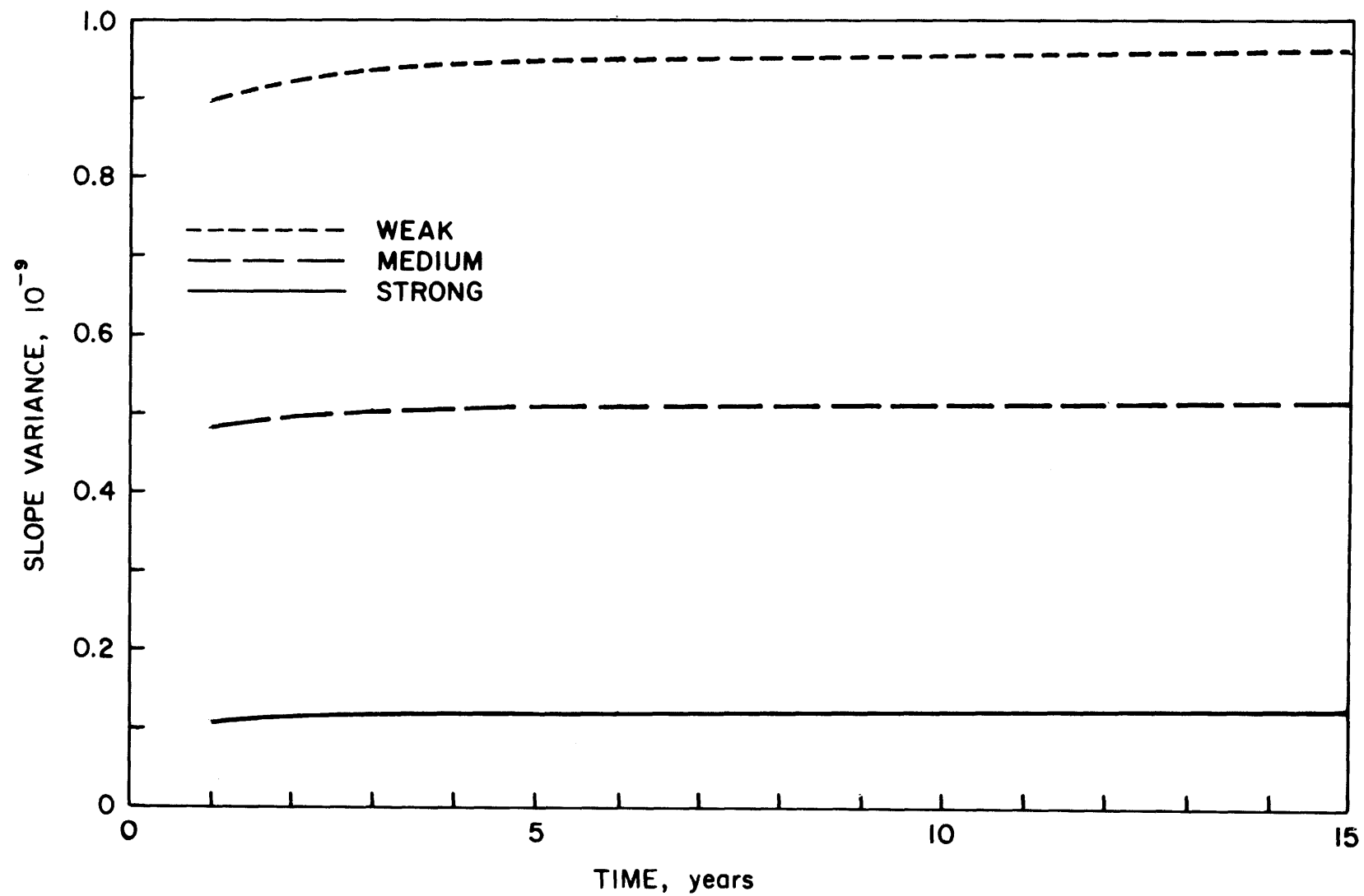


FIGURE 17. INFLUENCE OF MATERIAL PROPERTIES ON ROUGHNESS.

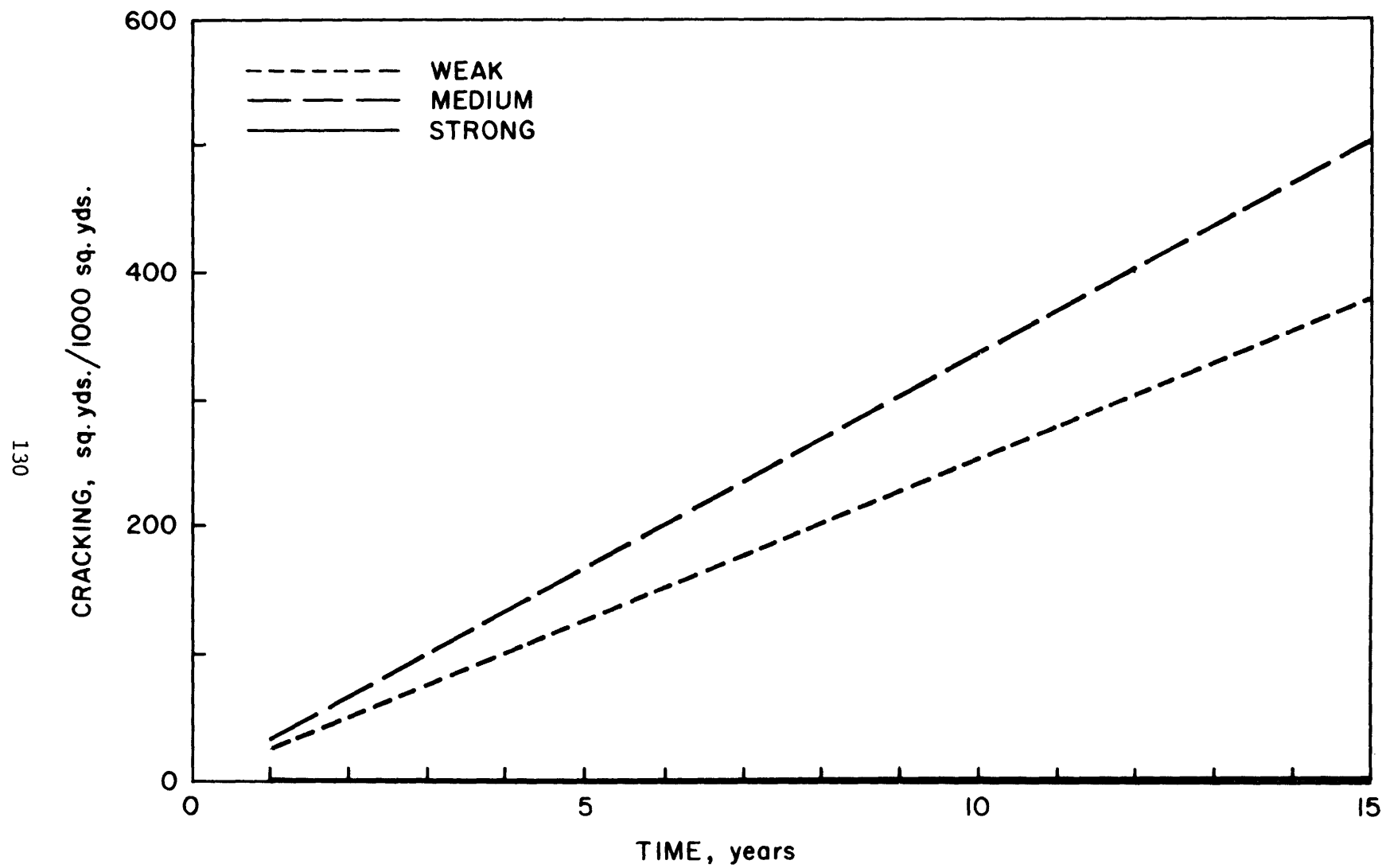
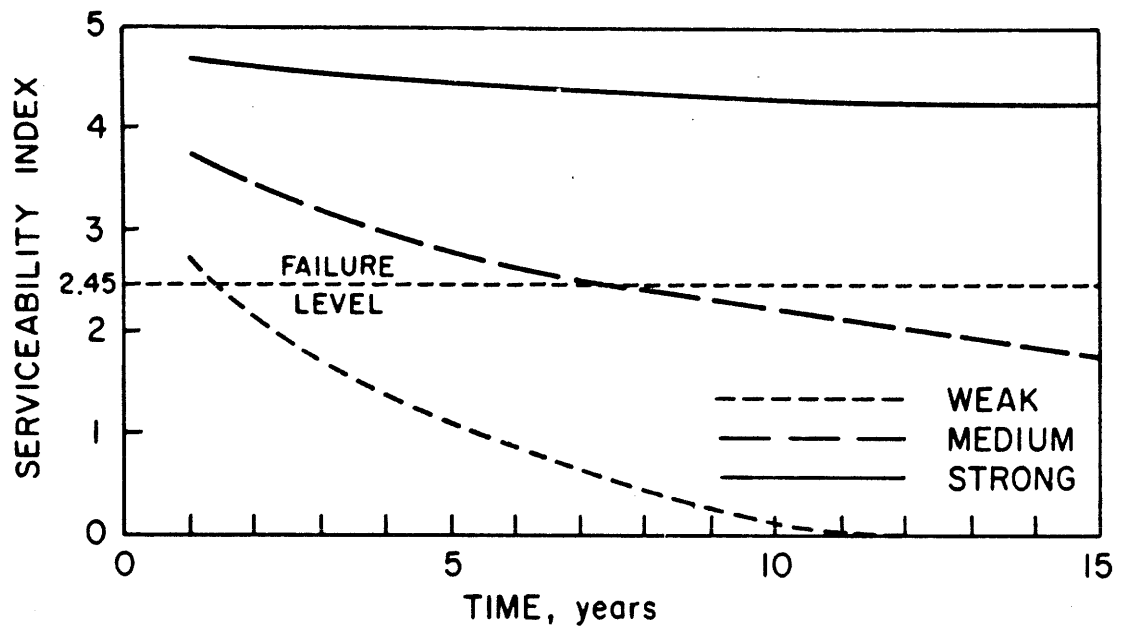
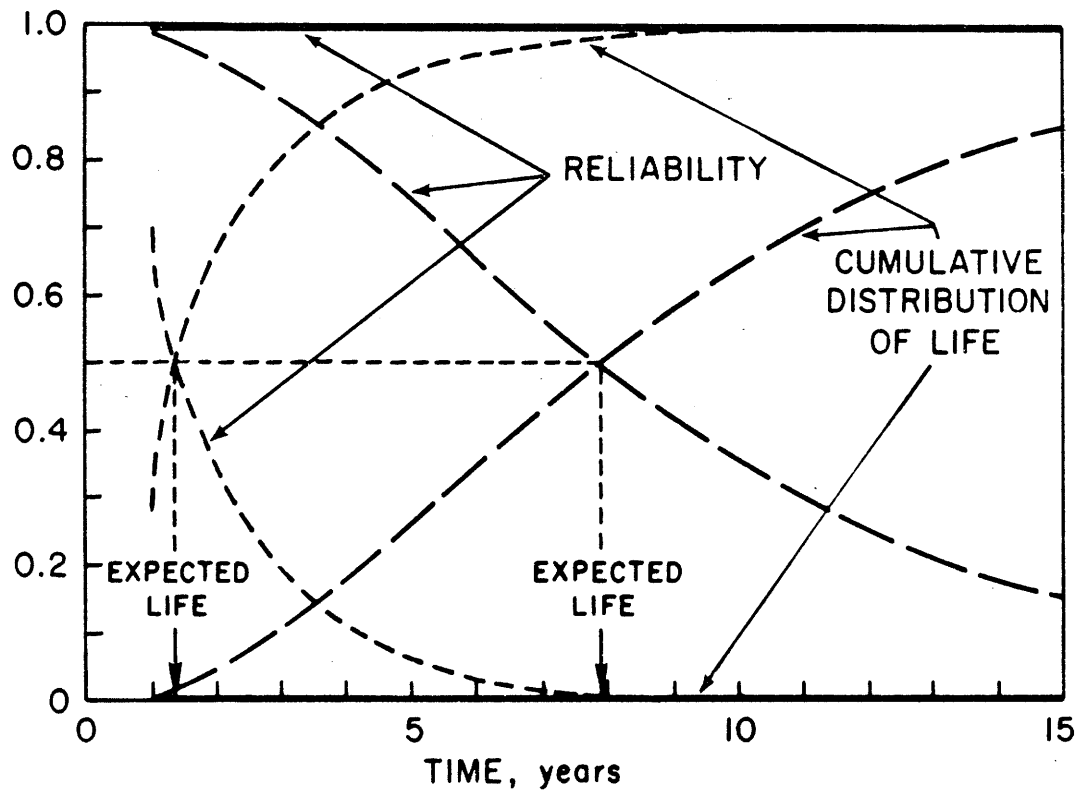


FIGURE 18. INFLUENCE OF MATERIAL PROPERTIES ON CRACK PROGRESSION.



(a)



(b)

FIGURE 19. INFLUENCE OF MATERIAL PROPERTIES ON (a) SERVICEABILITY INDEX, AND (b) RELIABILITY AND LIFE EXPECTANCY.

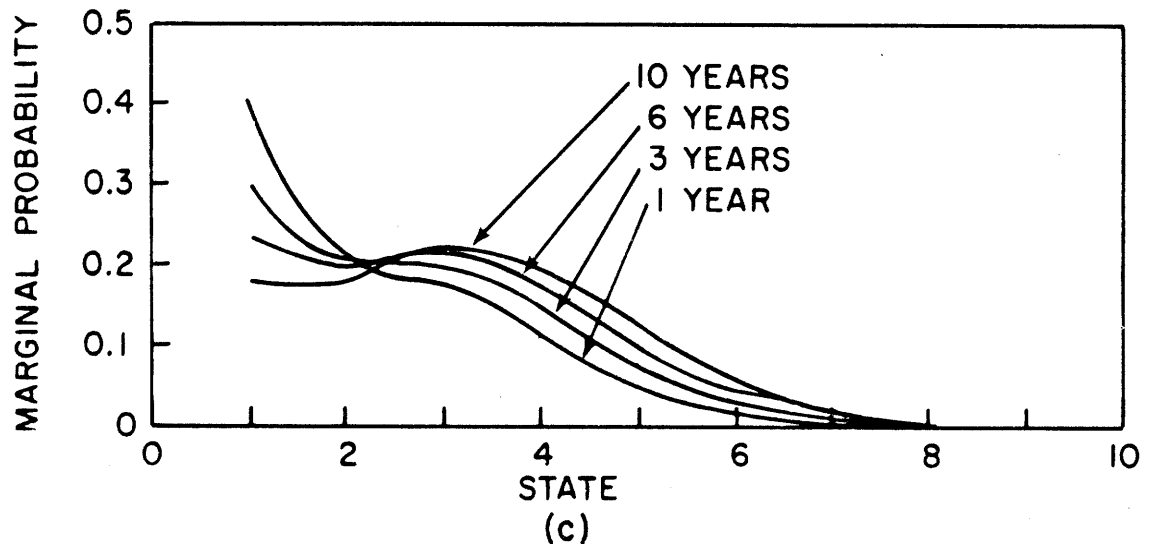
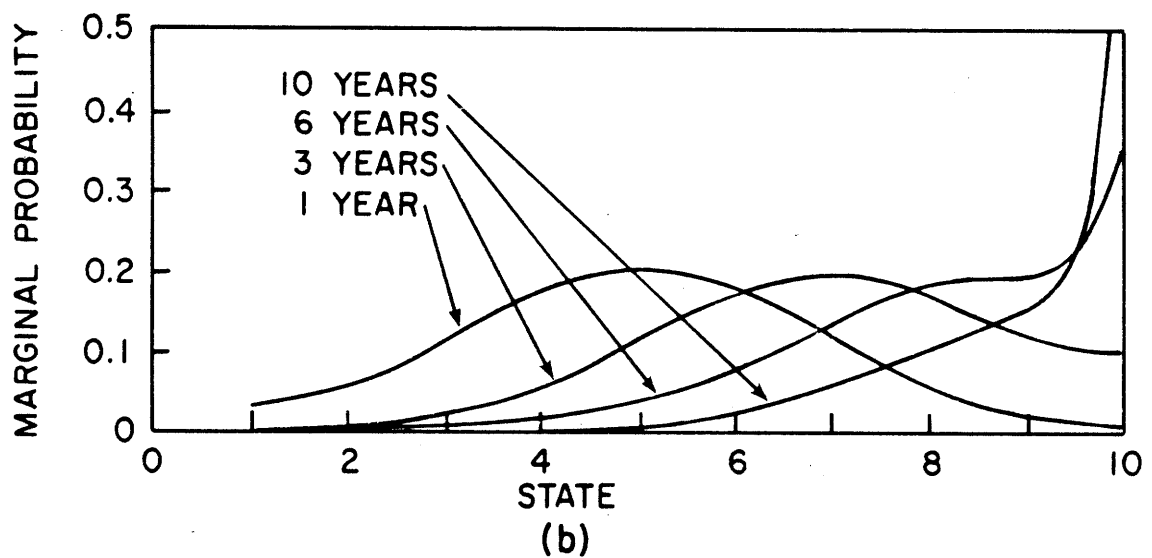
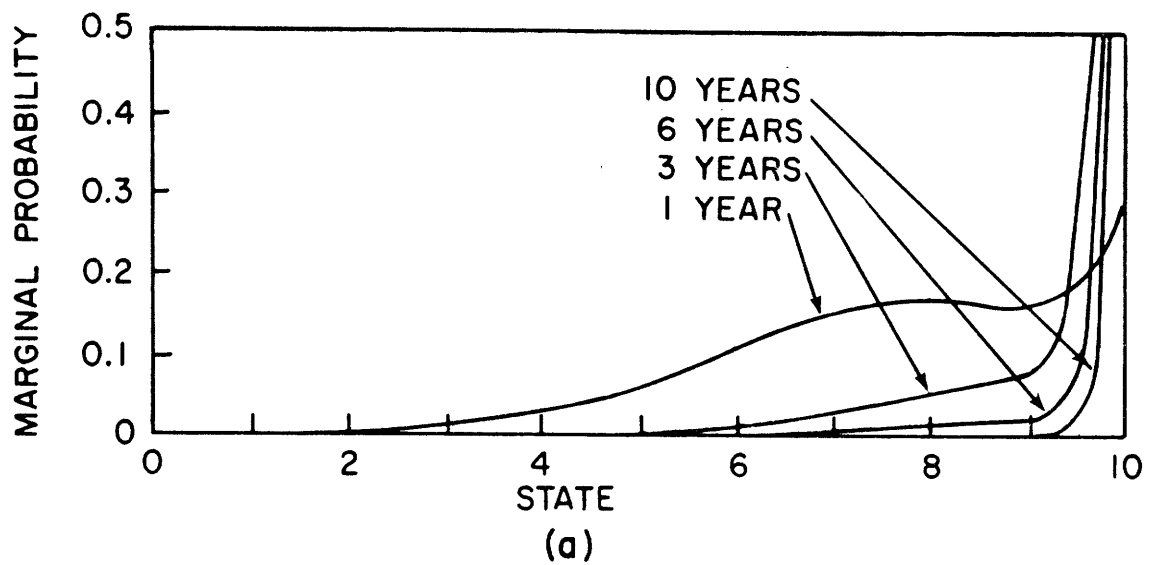


FIGURE 20. MARGINAL PROBABILITIES FOR (a) WEAK, (b) MEDIUM, AND (c) STRONG MATERIAL PROPERTIES.

field and experimental data, and hence a more realistic output is attained.

The marginal probabilities at different times are also presented in figures (20a) through (20c). In these figures it is shown that the probabilities of being in states of higher serviceability decrease with the passage of time. These probabilities are also smaller for the weaker systems than their corresponding values for the stronger systems. This again is in agreement with what is normally observed and anticipated in the real world systems.

IV.3.3. Quality Control Levels

In general, the uncertainties in the behavior of systems may be attributed to two sources: exogenous or uncontrollable sources, and endogenous or partially controllable sources. The former is related to uncertainties in loading patterns, both in time and space, and climatic environment. The latter, on the other hand, is associated with inherent variabilities in the physical properties of the system's constituents. These variabilities are a result of the mixing, placing, compaction, and testing operations of the materials in the system. Various levels of control can be exercised at each of these fabrication stages to obtain certain degrees of homogeneity and uniformity in the final product.

This section aims at examining various levels of controls on the fabrication of materials, and the resulting

influence on the future performance of the system, and its reliability and life expectancy. Since the level of quality control on system fabrication affects the statistical scatter of the constituent materials, each control level may be described in terms of this scatter. System 1 has been chosen for this study with three different quality control levels, defined as follows:

Quality Control Level	Statistical Scatter* in System's Properties	Statistical Scatter in Fatigue Properties
Poor	50%	40%
Medium	25%	20%
Good	10%	10%

Table (4.1) represents the inputs and outputs for the medium control case, while Tables (4.6) and (4.7) present the "poor" and "good" cases, in that order. In addition to the differences in the statistical scatter in the system's properties, the statistical properties of load characteristics are also different for the three cases at hand. These are listed as follows:

Quality Control Level	Scatter in Load Duration	Scatter in Load Amplitudes
Poor	60%	50%
Medium	25%	25%
Good	20%	10%

* This statistical scatter is defined in terms of the coefficients of variation for the system's properties.

Tables (4.1), (4.6), and (4.7) show that the expected values of rut depth are insensitive to the control levels, while the corresponding variances are significantly affected by these control levels. This may be due to the fact that rutting is attributed to channelized load effects. Apparently the changes in the statistical scatters of load amplitudes and duration are of little influence on the expected values of rut depth, while these changes as well as changes in the materials' properties influence the variances of the rut depth, which are measures of its statistical scatter.

The roughness of the system is extremely sensitive to the levels of quality control. As shown in the above tables and in Figure (21), the slope variance of the system increases as the statistical scatter in the system's properties increases. It is observed that the slope variance of the medium control case is roughly 6 times that of the good control case. This corresponds to the square of the ratio of their coefficients of variation; i.e. $(\frac{0.25}{0.10})^2 = 6.25$. Similarly the slope variance of the poor control case is approximately 4 times that of the medium control case, and 25 times that of good control case. These numbers also correspond to the squares of the ratios of the coefficients of variations for these cases, which are:

$$\frac{\text{Poor}}{\text{Medium}} = \left(\frac{0.50}{0.25}\right)^2 = 4$$

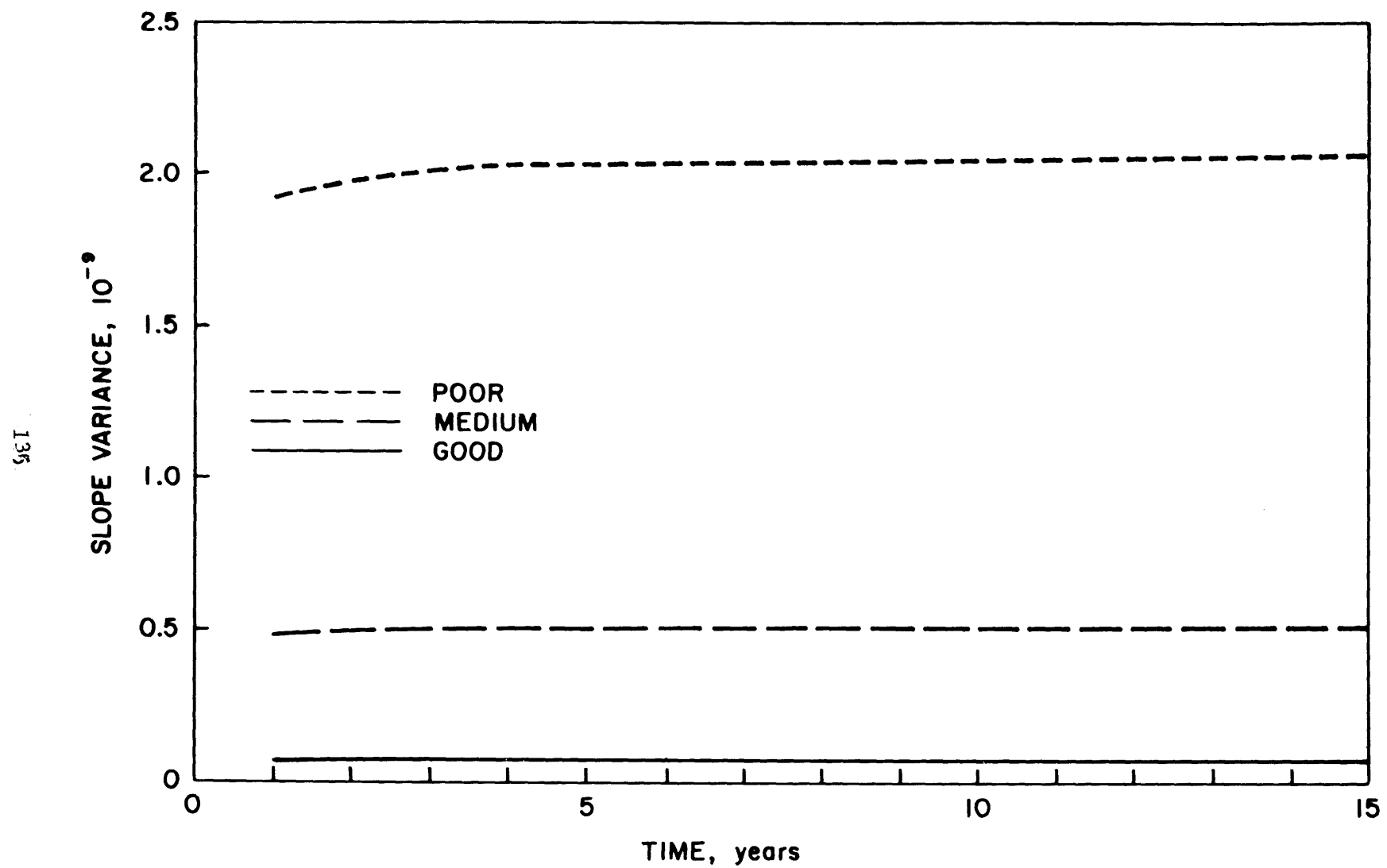


FIGURE 21. INFLUENCE OF QUALITY CONTROL LEVELS ON ROUGHNESS.

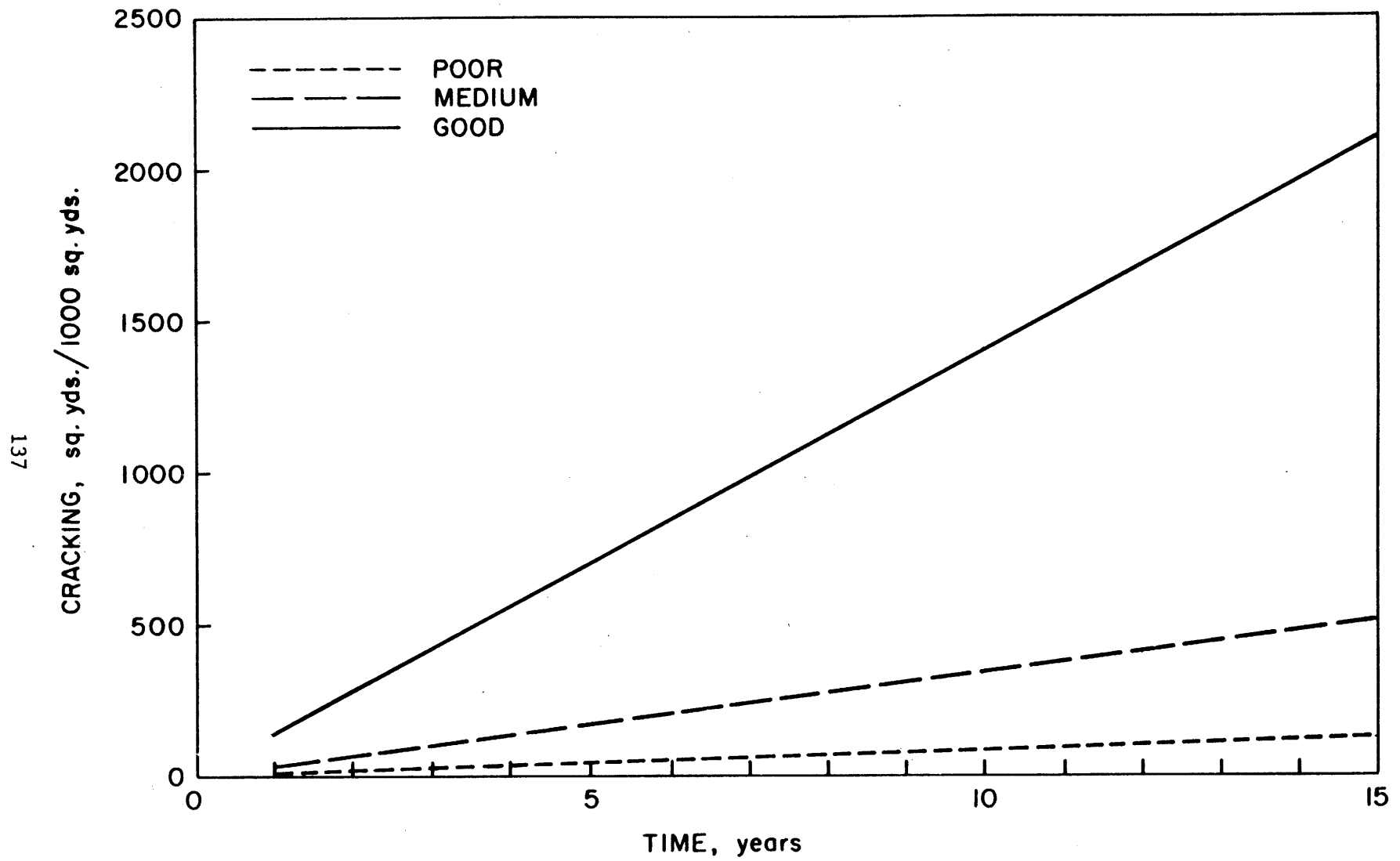


FIGURE 22. INFLUENCE OF QUALITY CONTROL LEVELS ON CRACK PROGRESSION.

$$\frac{\text{Poor}}{\text{Good}} = \left(\frac{0.50}{0.10} \right)^2 = 25$$

This observation can be readily seen from equation (2.30) in Chapter II, where the expected values of the slope variance are directly related to the variance of the materials' properties, i.e. σ_{η}^2 . Since the expected values of these properties are kept the same for the cases at hand, the above observation is analytically explained, and may be used as a general rule to compare the effects of quality control levels on the roughness of the system.

Figure (22) presents a comparison for the effects of quality control levels on the progression of cracks within the system. This figure shows that as the quality control level is increased, i.e. as the statistical scatter is reduced, the predicted expected values and variances of cracking are consistently increased. This is rather surprising, since it shows that the system completely cracks after about 8 years while it has a somehow low expected cracking area for the poor control level at the same time period. The variances for cracking of these systems at the corresponding time points are different however. The good control case resulted in a coefficient of variation which approximately 2.5 times that of the poor control case. This observation appears to be rather counter-intuitive, since it shows that the expected damage in the system is less for a larger spread in the materials' properties. In order to clarify this point further, and to trace the possible sources of discrepancy, further numerical analysis is conducted by isolating the different param-

meters involved. In defining the three different control levels earlier, two statistical scatters were varied: one related to the reponse characteristics, i.e. strains and deflections, and the other related to the fatigue properties. In the study of that particular case, it was assumed that both properties are positively correlated such that a larger scatter in one property is associated with a corresponding larger scatter in the other, and vice versa. Further, it was assumed that the two fatigue coefficients K_1 and K_2 in equation (4.1) were uncorrelated. In order to isolate the effects of all these factors, two sets of sensitivity analysis were conducted. In the first set, the spread in the fatigue coefficients was kept constant at a 20% coefficient of variation, while the spread in the strains was specified at 10%, 25%, and 50% coefficients of variation. The objective of this test was to examine the influence of changes in the scatter of the strains on crack progression. A comparison of the results shows that the changes in expected cracking due to changes in the spread of tensile strains was not very significant at that particular spread in the fatigue coefficients. These results are tabulated below. From the results, it is also shown that the amount of cracking is reduced rather significantly when the spread in the fatigue coefficients is increased. This is because of the exponential nature of Miner's rule which is presented in Chapter II.

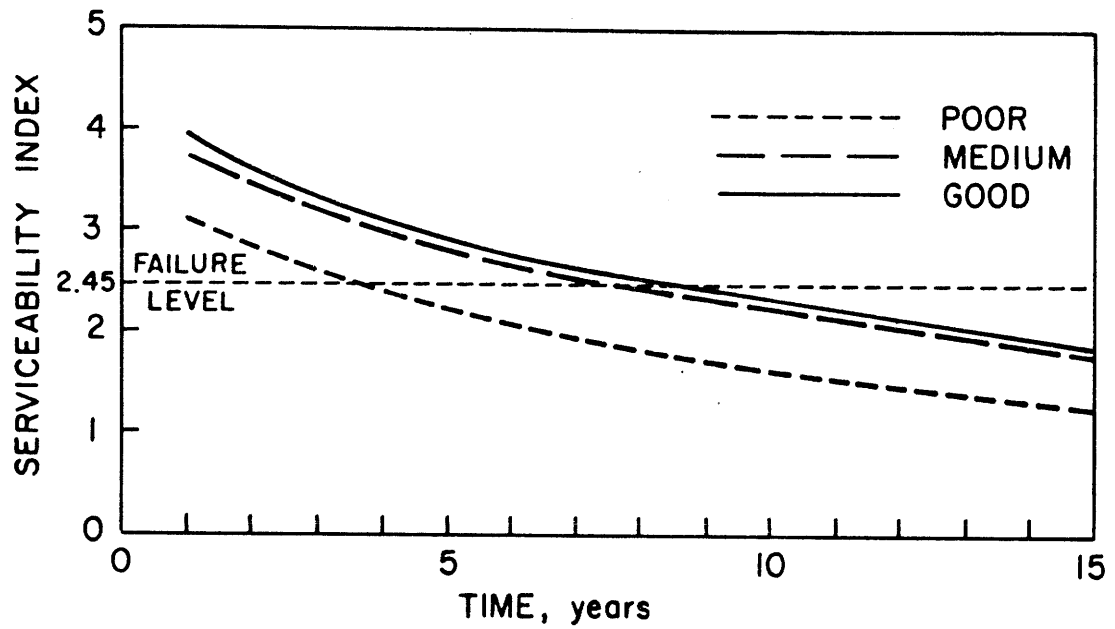
The second set of analysis concerning these results was intended to examine the influence of negative correlations which can exist between the strains and the fatigue coefficients, and between the fatigue coefficients themselves. For this purpose, a negative perfect correlation was assumed between the fatigue coefficients. Further, it was assumed that

the strains are negatively correlated to the fatigue coefficients, such that a 50% variation in the strains are associated with a 10% variation in the fatigue properties. Similarly, 25% and 10% variations in the strains are associated with 20% and 40% corresponding variations in the fatigue properties. The results indicate that for the negative correlation case, the increase in the statistical spreads of the strains results in a systematic reduction of both the expected values and the variances of cracking. These results, along with the previous ones, are presented in the following table, each measured after a one-year period. Clearly, the nature of the correlations which may exist among the various parameters requires a further investigation which must involve a number of fatigue experiments, in order to provide a proper understanding and sufficient information about this part of the model.

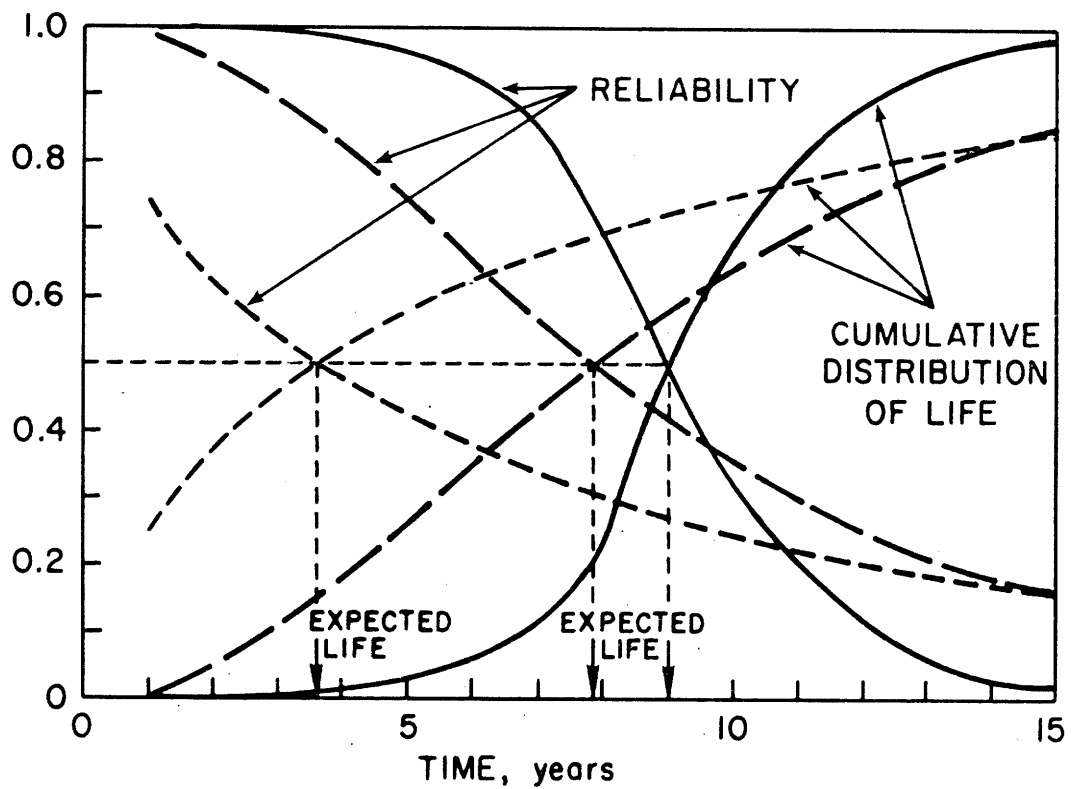
Scatter in Materials Properties	Scatter in Fatigue Properties	Expected Fatigue Cracking After a One year Period
50%	10%*	105.2*
	20%	31.5
	40%	8.3
25%	20%*	35.6*
	20%	33.6
10%	10%	140.1
	20%	34.3
	40%*	8.9*

Cracking, however, is not a dominating factor in AASHO's serviceability equation, and therefore its contribution to the changes in

* These values correspond to perfect negative correlation of fatigue coefficients.



(a)



(b)

FIGURE 23. INFLUENCE OF QUALITY CONTROL LEVELS ON (a) SERVICEABILITY INDEX, AND (b) RELIABILITY AND LIFE EXPECTANCY.

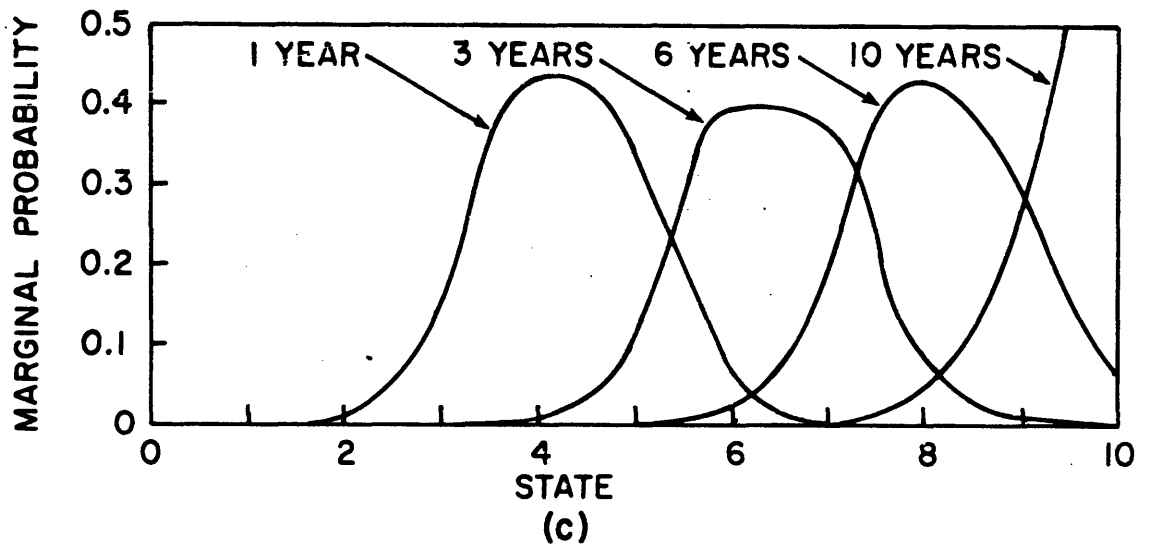
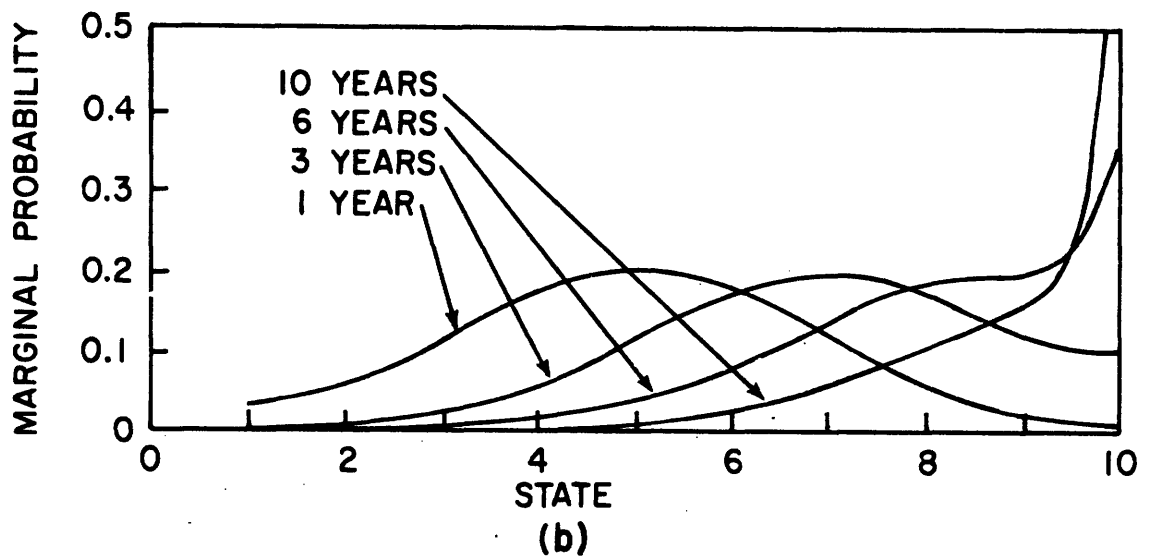
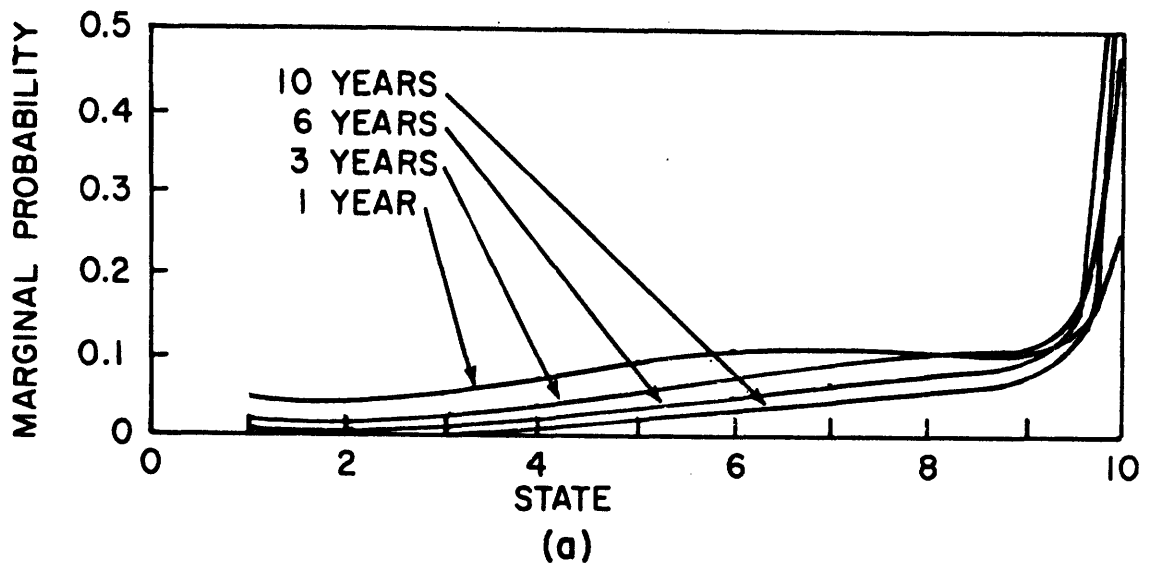


FIGURE 24. MARGINAL PROBABILITIES FOR (a) POOR, (b) MEDIUM, AND (c) GOOD QUALITY CONTROL LEVELS.

serviceability levels is of less importance than the rut depth and the slope variance. This is shown in figure (23a) where the serviceability of the systems at hand increase as the quality control levels are increased. Similarly, the reliability of the systems at any time point and the life expectancy are higher for higher control levels, as shown in figure (23b). In this figure, the lower reliability values for the system with higher control levels when its serviceability index reaches the failure level can be attributed to the smaller statistical scatter in the serviceability of the system. This scatter is represented by the marginal state probabilities which are plotted in Figure (24) for the three cases at hand.

The above sensitivity analysis has been of a rather limited scope, but it clearly shows that the methodology can be expanded into a large scale sensitivity analysis in which a variety of factors can be examined. From such a study, one can determine the influence of each of these factors and the interactions among several factors that affect the behavior of the system in this complex operational environment.

IV.4. Summary

The preceding sections have been concerned with the applications of the structural and serviceability models developed earlier in Chapter II. Further, several systems have been analyzed to demonstrate the capability of these models in providing information about the behavior of the systems and their sensitivity to various design factors under random load and temperature histories. The following factors have been investigated in this study:

- A. Influence of Geometry: It was observed that the thicker the layers of the system become, the higher are its serviceability, reliability and life expectancy.
- B. Influence of Materials' Properties: The results of this study showed that stronger properties result in lower damage rates, and consequently higher serviceability, reliability and life expectancy.
- C. Influence of Quality Control Levels: It was shown that higher quality control levels exercised on systems fabrication generally result in higher serviceability, reliability, and life expectancy.

In this context, it was observed that the slope variance is approximately proportional to the square of the spatial coefficients of variation of the materials' properties. This implies that when the coefficient of variation

of one system is twice that of second system, the roughness of the former is approximately four times that of the latter. In addition, it was observed that both the predicted expected value and variance of cracking increase as the quality control becomes higher. This is attributed to the decrease in the expected values of the number of load cycles required to produce fatigue failure in the system for higher control levels. This number is inversely proportional to both the expected value and variance of cracking.

In general, it may be stated that in all the above observations, the models provided reasonable agreements with the expected behavior of the real-world systems. While the numerical value provided may not be physically meaningful in some cases, the predicted trends in the behavior of the systems as well as in their reliability and life expectancy reasonably agree with the corresponding trends exhibited by the real-world systems as perceived by engineering experience and intuitive judgement.

Further refinement and calibration of the models may come about as these models are used in conjunction with laboratory and field observations, such as test track studies. In this case, the results of the model are continuously compared with those observed or measured, and necessary adjustments are made for the relevant coefficients and parameters of the model.

V. SUMMARY AND CONCLUSION

This study presented a framework for analysis and selection of optimal systems for highway pavements. The development of this framework is predicted upon the basic philosophy that public facilities are intended to provide certain services to their users. Thus, the functioning of these systems must be evaluated from the standpoint of the users' demand and satisfaction.

To this end, a set of models has been developed to account for the interactions which exist among the materials, environment and economic attributes. The analysis and selection process is realized through the implementation of three major phases. One is concerned with the selection of materials and probabilistic evaluation of the physical behavior of the system in a simulated operational environment, utilizing a set of mechanical and phenomenological models. The second phase is aimed at the evaluation of measures of effectiveness for the system at hand in terms of its serviceability, reliability, and the maintenance strategies throughout the design lifetime. Phase three addresses itself to the system's management issues in terms of choice of optimal maintenance policies and decisions related to alternative design tradeoffs and cost optimization.

In order to demonstrate the capacity of these models to predict the physical behavior of the system and its performance characteristics, a set of numerical examples

are presented. These examples examined the sensitivity of the behavior of the system to various design parameters. In this context, the geometry of the system, its physical properties, and the level of quality control are studied in terms of their influence on the system's serviceability, reliability, and life expectancy. These studies showed that the trends predicted by the model are in reasonable agreement with the anticipated behavior of the real-world systems based on experience and engineering judgement. Further, a simple illustration for the selection of optimal maintenance policies using the dynamic programming techniques and decision analysis is presented.

The basic features which characterize this study are summarized as follows:

1. The design framework proposed in this study represents a departure from the conventional methods which are generally pursued in the literature of structural design. Instead, design is viewed as a process of sequential evolution of systematic analysis whose ultimate goal is the achievement of an optimal design configuration suitable for a given set of goals and constraints.
2. The criteria for system selection and evaluation are based on the users' subjective preferences for the systems as derived from their particular needs and set of values. From this standpoint, the highway pavement is viewed as a system which is providing certain services to its users. The

quality of providing these services at any time must then be evaluated from the users' preferences and satisfaction.

3. The proposed models cover a wide spectrum of activities which encompass a rather larger body of knowledge, ranging from rational mechanics to probability and operations research disciplines. This wide coverage provides a means of continuity and integrity to the design process. For example one can study the influence of changes in geometry, physical properties, traffic patterns, or quality control levels not only the future behavior of the system, but also on its life, maintenance policies, costs, and so forth, through a relatively direct process.
4. The models possess a causal structure which defines the interactions among the system, the operating environment, and the imposed economic constraints. Further the feedback processes resulting from maintenance activities are accounted for.
5. The models recognize and incorporate elements of uncertainties which are inherent in both the physical properties of the system and surrounding environment.

VI. RECOMMENDATIONS FOR FUTURE WORK

In view of what has been presented so far, further research activities in this area can proceed along two lines, not necessarily mutually exclusive. The first involves field verification and calibration of the models, while the second includes the extension of the existing models within the established framework. In this chapter, these activities are discussed with emphasis on the relevance and applicability of the models, and their adaptability to a comprehensive design methodology for highway systems.

VI.1. Field Verification and Model Calibration

In order that the models are used as a meaningful design tool for highway pavements, they must be tested and calibrated against actual field and laboratory measurements. Test tracks and accelerated-life experiments provide some means for these measurements. In this context, the particular values predicted by the models must be compared with the measured values in field. If significant discrepancies exist, appropriate adjustments both in the particular relationships and the relevant assumptions must be made accordingly.

Both laboratory experiments and field observation may be used to examine the validity of several of the assumptions upon which the model development is based, and to provide a proper characterization for the in situ materials. Such tests must include the range of linearity for the characteristics

of the system under representative loading and temperature histories. For these, one may assess the errors involved in linear approximations, and the significance of these approximations in the prediction of the system response.

Pavement materials are generally known to exhibit different behavior in the unloading cycles than during the loading cycles. This means that the recovery portion of the load-deflection curve is not a "mirror image" of the loading portion, and usually a permanent deformation results from each load cycle. This represents a non-linear behavior to which the response solution under repeated loading conditions must be adjusted. This explains why the values of rut-depth and roughness obtained in Chapter IV were extremely low. As it stands now, the model allows a complete recovery after long periods of time when the loads are removed, contrary to experimental observations.

Further characterization requirements involve the determination of the coefficients for the time-temperature superposition of the system response within realistic range of values for temperatures and material properties.

The fatigue model used in this study is based on a phenomenological approach, namely the Miner's criterion. This approach however, does not provide a quantitative description for crack initiation and progression, nor does it account for the viscoelastic nature of these processes. A more realistic approach is needed to account for the

different stages of crack formation and accumulation in the system based on a micromechanics method of approach. In this regard, a stochastic model utilizing the knowledge of fracture mechanics can be developed for the three-layer viscoelastic system. At this stage, a random walk model [F2] used in conjunction with a fracture mechanics method may provide a more realistic substitute for Miner's criterion, to account for random initiation and growth of cracks in the system.

A final suggestion with regard to refinement of the existing models is related to the static load solution. The simulation procedure developed for this solution based on the Monte Carlo method [F1] generally requires a lengthy computational time for any meaningful statistical analysis. To substantially reduce this computational time, a closed-form probabilistic analysis for the static load response is needed to obtain the statistical parameters necessary to describe this response stochastically.

VI.2. Extension of the Existing Models

The extension of the above models may proceed in parallel along the following lines:

A. Implementation of the maintenance decision framework with the aid of the computer to incorporate a large set of maintenance levels studied over extended periods of time. It was shown in Chapter III, that as the choice space

for maintenance levels at any time becomes larger, and as the period of study becomes longer, the aid of computers in the decision-making process becomes indispensable. The development of a computer algorithm for this purpose, however, should not constitute a major effort, since the formulation presented in Chapter III is structured in an algorithmic form readily adaptable for computer programming.

Once this is accomplished, a large-scale sensitivity analysis may be undertaken to identify the sensitivity of the measures of effectiveness of the system to various maintenance policies. Furthermore, the influence of the particular measures of effectiveness on the choice of optimal strategies for maintenance also can be examined. This can provide an insight into the selection of relevant measures for maintenance optimization as a part of an overall system optimization framework.

B. Study of potential integration of the Cost Model with the existing models. This model has been developed primarily for low volume traffic roads, but the possibility of using it for normal highway networks has been explored. The cost model generates the cost of construction of a given design configuration, and determines a set of maintenance and vehicles operation costs for various maintenance strategies. These costs can then be used in conjunction with the optimization criteria for the selection of an optimal system.

C. Establishment of a total cost optimization framework: The total cost of a highway pavement, C_T , is viewed as the sum of the initial construction cost, C_1 , the annual maintenance costs, including resurfacing, C_2 , and the vehicles operation, or users costs, C_3 . One can express the relationship as follows:

$$C_T = C_1 + C_2 + C_3 \quad (6.1)$$

The maintenance and vehicle's operation cost are discounted to their net present worth in the above equation.

It is assumed here that the design of a highway involves an optimal resource allocation suitable for a certain set of goals and constraints. The goals can be translated into economic terms, such as achieving acceptable performance to the users of the facility at a minimum cost. The achievement of an acceptable performance represents a constraint on minimum cost. Without this constraint, one may use the extreme argument that the cheapest way to build a highway is not to build one at all. In addition, there may be other constraints on cost, resulting from the availability and distribution of funds for a given locality. For example, it may prove more economically feasible to initially build a strong highway and allocate less funds for future maintenance. However, the particular strategy may not be adopted when sufficient funds are not available for this purpose,

which would consequently require a less economical alternative. Similarly, when federal funds are available for construction costs alone, local communities may prefer to allocate the maximum possible funds for construction, while spending less for future maintenance which, in this case, is funded by local taxes.

In general, one may express this cost optimization problem in the following manner:

$$\text{Min. } C_T = \sum_{i=1}^3 a_i C_i \quad (6.2)$$

subject to the constraints:

$$\sum_{i=1}^3 d_{ij} C_i \leq b_j, \text{ for } j = 1, \dots, m \quad (6.3)$$

and

$$C_1 \geq 0, C_2 \geq 0, C_3 \geq 0 \quad (6.4)$$

In the above equation C_T is referred to as the objective function, which is the overall measure of effectiveness for the system. a_i represents the increase in C_T for each unit increase of the corresponding cost component C_i . b_j is the available resource or fund for the relevant cost components. For example if maintenance cost and vehicle operation cost are not to exceed a certain amount Z , one can state that:

$$C_1 + C_2 \leq Z \quad (6.5)$$

The left side of the inequalities (6.3) represents the total cost resulting from the relevant activities in these inequalities.

Equations (6.2) through (6.4) represent a general form of a large class of optimization problems identified as the linear programming problem. This solution of this class of this class of problems is obtained using the simplex method, which is an iterative algorithmic method whose objective is the achievement of a set of optimal resources to minimize or maximize an objective function [H3, H6, W3].

The development of this structure is necessary to attain an overall design methodology in which various strategies and design configurations are generated and evaluated, from which an optimal design is provided.

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VIII. BIOGRAPHY

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He attended the University of Baghdad in 1962 and graduated with a Bachelor of Science with distinction in Civil Engineering in 1966. He then joined the Iraqi Air Force as a Second Lieutenant Engineer. In this capacity, he planned and supervised construction and maintenance of airfields and other airport facilities. Upon completion of his military service, he worked with private contractors and consultants as a project manager and field engineer for three public projects of costs totalling over 10 million dollars.

In 1969, the author won a Goulbenkian Fellowship jointly sponsored by the Goulbenkian Foundation in Lisbon, Portugal and the National Foundation for Scientific Research in Baghdad, Iraq. He enrolled in the Graduate School of Engineering at the Massachusetts Institute of Technology, where he obtained his S.M. degree in Civil Engineering in 1971. He has been a Research Assistant in the Department of Civil Engineering since June 1969. In June 1971 he became a research staff member of the M.I.T., Division of Sponsored Research, consulting for the Office of Research, Federal Highway Administration, U.S. Department of Trans-

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He is graduating with an Sc.D. in Civil Engineering in September 1972.

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IX. APPENDICES

APPENDIX I

DETAILED STOCHASTIC SOLUTIONS OF THE MODEL

The following convolution integral represents the response of the system to load and environment:

$$R(t) = \int_0^t F(t, \tau, \phi) \frac{\partial P(\tau)}{\partial \tau} d\tau \quad (1)$$

This expression can be expanded into (S2):

$$R(t) = \int_0^t \beta[\phi(t)] \sum_{i=1}^N -G_i e^{-t \delta_i} A(\tau) \frac{\sinh\left(\frac{\gamma(\tau) \delta_i D(\tau)}{2}\right)}{1 + \left(\frac{\delta_i \gamma(\tau) D(\tau)}{2\pi}\right)^2} d\tau \quad (2)$$

The above integral can be broken down into a sum of integrals of the following form:

$$R(t) = \sum_{k=1}^L \int_{t_{k-1}}^{t_k} \beta[\phi(t_L)] \sum_{i=1}^N -G_i A_k(\tau) \frac{\sinh\left(\frac{\gamma_k \delta_i D_k(\tau)}{2}\right)}{1 + \left(\frac{\gamma_k \delta_i D_k(\tau)}{2\pi}\right)^2} \exp[-\delta_i \{(t_k - \tau) + \sum_{p=k+1}^L \gamma_p t_{DEL}\}] d\tau \quad (3)$$

Traffic loads are assumed to arrive as a Poisson process.

Equation (3) above can then be written as:

$$R(t) = - \sum_{k=1}^L \sum_{j=1}^{n_k} \sum_{i=1}^N \beta[\phi(t_L)] G_i A_{jk} \frac{\sinh\left(\frac{\gamma_k \delta_i D_{jk}}{2}\right)}{1 + \left(\frac{\gamma_k \delta_i D_{jk}}{2\pi}\right)^2}$$

$$\exp[-(\gamma_k t_{\text{DEL}} \delta_i + \sum_{p=k+1}^L \gamma_p t_{\text{DEL}} \delta_i)] \exp(\tau_j \gamma_k \delta_i)$$

where: D_{jk} - is the j th load duration in the k th period

n_k - is the number of Poisson arrivals of loads in the k th period.

$t_{\text{DEL}} - (t_k - t_{k-1}) = \text{constant}$

γ_k - represents shifting factors for the response as a function of temperature changes. Temperatures are taken as averages and are constant over each k .

Let: $V_{ik} = \exp[-(t_{\text{DEL}} \delta_i \sum_{p=k}^L \gamma_p)]$

$$\theta_{ik}(\tau_j) = \exp[\tau_j \gamma_k \delta_i]$$

$$S_{ik} = \gamma_k \delta_i$$

Equation (4) above becomes

$$R(t) = \beta[\phi(t_L)] \sum_{k=1}^L \sum_{j=1}^{n_k} \sum_{i=1}^N G_i A_j V_i \theta_i(\tau_j) \frac{\sinh(\frac{S_{ik} D_{jk}}{2})}{1 + (\frac{S_{ik} D_{jk}}{2\pi})^2} \quad (5)$$

The materials variables G_i represent the characteristic function of the system, which in this case describes the response of the system to a step load applied in a static fashion. These variables can be expressed as:

$$G_i = \eta \bar{G}_i \quad (6)$$

where: \bar{G}_i - is the mean value of G_i

η - is a random variable with a mean of 1.0, and a variance σ_η^2

Define:

$$R_k(t) = \sum_{j=1}^{n_k} A_{jk} \eta \sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik} (\tau_j) \frac{\sinh\left(\frac{D_{jk} S_{ik}}{2}\right)}{1 + \left(\frac{S_{ik} D_{jk}}{2\pi}\right)^2} \quad (7)$$

Where $R_k(t)$ represents the contribution of the k th period to the response at time t .

Equation (5) becomes:

$$R(t) = -\beta[\phi(t_L)] \sum_{k=1}^L R_k(t) \quad (8)$$

where

$$\beta[\phi(t_L)] = \frac{T(t_L)}{T_0} = \beta_L \quad (9)$$

where: $T(t_L)$ - temperature at time t

T_0 - reference temperature

The first and second moments of $R(t)$ may be written

as:

$$\begin{aligned} E[R(t)] &= -\frac{T(t)}{T_0} E\left[\sum_{k=1}^L R_k(t)\right] \\ &= -\frac{T(t)}{T_0} \sum_{k=1}^L E[R_k(t)] \end{aligned} \quad (10)$$

$$\begin{aligned}
\text{Var}[R(t)] &= \frac{T(t)^2}{T_0^2} \text{Var}\left[\sum_{k=1}^L R_k(t)\right] \\
&= \frac{T(t)^2}{T_0^2} \left\{ \sum_{k=1}^L \text{Var}[R_k(t)] \right. \\
&\quad \left. + 2 \sum_{k=1}^L \sum_{\ell=k+1}^L \text{Cov}[R_k(t), R_\ell(t)] \right\} \quad (11)
\end{aligned}$$

where: $E[\quad]$ - expected value of $[\quad]$

$\text{Var}[\quad]$ - variance of $[\quad]$

$\text{Cov}[\quad]$ - covariance of $[\quad]$

In equation (7) above, $R_k(t)$ is a filtered Poisson process of the general form:

$$R_k(t) = \sum_{\tau} w(t, \tau, y)$$

The general solution for this type of process has been provided by Parzen [Pl] in the following forms:

$$E[R_k(t)] = \lambda \int_0^t E[w_k(t, \tau, y)] d\tau \quad (12)$$

$$E \text{Var}[R_k(t)] = \lambda \int_0^t E[w_k(t, \tau, y)^2] d\tau \quad (13)$$

$$\begin{aligned}
\text{Cov}[R_k(t), R_m(s)] &= \lambda \int_0^t E[w_m(s, \tau, y) \\
&\quad w_k(t, \tau, y)] d\tau \quad (14)
\end{aligned}$$

where: λ - is the Poisson rate of arrival of loads

$y = f(D, A)$

Combining equations (7) and (12), we get

$$E[R_k(t) | \eta = \eta_0] = \eta_0 \lambda \int_0^{t_{DEL}} \sum_{i=1}^N E[\bar{G}_i A_k V_{ik} \theta_{ik}(\tau) \frac{\sinh(\frac{S_{ik} D_k}{2})}{1 + (\frac{S_{ik} D_k}{2\pi})^2}] d\tau \quad (15)$$

Where $E[\quad]$ signifies that the expectation is conditional on η (i.e., for a fixed value of η , η_0). We assume that A_k and D_k are not correlated, which implies that the load amplitude and velocity are virtually uncorrelated. This may be justified by the argument that in the present interstate systems, large trucks and busses travel at fairly high speeds, while smaller cars may not be able to do so. If there still is correlation, it may be neglected for simplification of the problem at hand.

Equation (15) above becomes:

$$E[R_k(t) | \eta] = \eta_0 \lambda E[A_k] \int_0^{t_{DEL}} \sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik}(\tau) E\left[\frac{\sinh(\frac{S_{ik} D_k}{2})}{1 + (\frac{S_{ik} D_k}{2\pi})^2}\right] d\tau \quad (16)$$

let:

$$\phi_{ik} = \left[\frac{\sinh(\frac{S_{ik} D_k}{2})}{1 + (\frac{S_{ik} D_k}{2\pi})^2} \right] \quad (17)$$

and

$$\bar{\phi}_{ik} = E[\phi_{ik}] \quad (17a)$$

In equation (16), we have

$$\begin{aligned} \int_0^{t_{DEL}} \theta_{ik}(\tau) d\tau &= \int_0^{t_{DEL}} e^{(\tau \gamma_k \delta_i)} d\tau \\ &= \frac{1}{\gamma_k \delta_i} [e^{(t_{DEL} \gamma_k \delta_i)} - 1] \end{aligned} \quad (18)$$

Let:

$$w_{ik} = \frac{1}{S_{ik}} \{ \exp(S_{ik} t_{DEL}) - 1 \} \quad (18a)$$

Taking the expectation on η and substituting for equation (18a) in equation (16), we get:

$$E[R_k(t)] = E_\eta [\eta \lambda \bar{A}_k \sum_{i=1}^N \bar{G}_i V_{ik} \bar{\phi}_{ik} w_{ik}] \quad (19)$$

$$E[R_k(t)] = \bar{\eta} \lambda \bar{A}_k \sum_{i=1}^N \bar{G}_i V_{ik} \bar{\phi}_{ik} w_{ik} \quad (20)$$

in which $\bar{\eta} = 1.0$.

To find $\bar{\phi}_{ik}$, we resort to the following approximate method:

Given a function $y = g(x)$ we can get the first and/or second order approximation (and higher orders if so desired), by the use of the Taylor series expansion of $g(x)$ around the mean of x . Such as follows:

$$g(X) \approx g(\bar{x}) + (X - \bar{x}) \left. \frac{dg(x)}{dx} \right|_{\bar{x}} + \frac{(X - \bar{x})^2}{2} \left. \frac{d^2g(x)}{dx^2} \right|_{\bar{x}} + \dots \quad (21)$$

From this, the second order approximation of the expected value of Y, is found as follows:

$$E[Y] = E[g(X)] \approx E[g(\bar{x})] + E\left\{[X - \bar{x}] \left. \frac{dg(x)}{dx} \right|_{\bar{x}}\right\} + E\left\{\frac{(X - \bar{x})^2}{2} \left. \frac{d^2g(x)}{dx^2} \right|_{\bar{x}}\right\} + E[\dots] \quad (22)$$

The second term is zero and the equation becomes

$$E[g(x)] \approx g(\bar{x}) + \frac{1}{2} \sigma_x^2 \left[\left. \frac{d^2g(x)}{dx^2} \right|_{\bar{x}} \right] \quad (23)$$

Similarly, by taking the variance of equation (21) one can prove that the first order approximation of the variance of g(x) is:

$$\begin{aligned} \text{Var}[g(x)] &\approx \text{Var}\left[\left. \frac{dg(x)}{dx} \right|_{\bar{x}} (X - \bar{x})\right] \\ &\approx \left. \frac{dg(x)}{dx} \right|_{\bar{x}}^2 \sigma_x^2 \end{aligned} \quad (24)$$

The third term in equation (21) will yield in the variance an additional term of the form:

$$\left. \frac{d^2g(x)}{dx^2} \right|_{\bar{x}} \{\text{Var}[x^2] - 2 \bar{x} \sigma_x^2\}$$

This term may be added to equation (24). However, some studies indicated that for all practical purposes, the first order approximation for the variance determination is sufficient [B1].

Benjamin and Cornell [B1] state that this approximation holds if the function $g(x)$ is sufficiently well behaved around the value \bar{x} (the mean of x), and the coefficient of variation of x is not very large. The question of how large this coefficient of variation be depends upon the degree of nonlinearity of $g(x)$ in the region around \bar{x} and on the degree of acceptable approximation error.

The approximate solutions presented in equations (23) and (24) will be used frequently in what follows for the solution of the basic response, and at later stages for the solution of the damage accumulation using the modified Miner's law approach. Accordingly, $\bar{\phi}_{ik}$ in equation (17a) can be determined as follows:

$$\begin{aligned}
 E[\phi_{ik}(D)] &= \phi_{ik}(\bar{D}) + \frac{1}{2} \left[\frac{d^2 g(D)}{dD^2} \right] \sigma_{D_k}^2 \\
 &\approx \frac{\sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right)}{1 + \left(\frac{S_{ik} \bar{D}_k}{2\pi}\right)^2} + \frac{\sigma_{D_k}^2}{2} \left[\frac{\left(\frac{S_{ik}}{2\pi}\right)^2}{1 + \left(\frac{S_{ik} \bar{D}_k}{2\pi}\right)^2} \right. \\
 &\quad \left. \{ \pi^2 \left[1 + \left(\frac{S_{ik} \bar{D}_k}{2}\right)^2 \right] \sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right) - 2 S_{ik} \bar{D}_k \cosh\left(\frac{S_{ik} \bar{D}_k}{2}\right) \right. \right. \\
 &\quad \left. \left. - 2 \sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right) + \left(\frac{S_{ik}}{2\pi}\right)^2 D_k \frac{\sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right)}{1 + \left(\frac{S_{ik} \bar{D}_k}{2\pi}\right)^2} \right\} \right] \quad (25)
 \end{aligned}$$

$$\begin{aligned} \bar{\psi}_{ik} = & \frac{\sinh\left(\frac{S_{ik} D_k}{2}\right)}{1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2} + \frac{\sigma_D^2}{2} \frac{\left(\frac{S_{ik}}{2\pi}\right)^2}{1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2} \left\{ \sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right) \right. \\ & \left. \left[\pi^2 \left(1 + \left(\frac{S_{ik} \bar{D}_k}{2\pi}\right)^2\right) - 2 + \frac{\left(\frac{S_{ik}}{2}\right)^2 D_k^{-2}}{1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2} \right] \right. \\ & \left. - 2 S_{ik} \bar{D}_k \cosh\left(\frac{S_{ik} \bar{D}_k}{2}\right) \right\} \end{aligned} \quad (26)$$

$\bar{\phi}_{ik}$ can be substituted back into equation (20) to determine the value of $E[R_k(t)]$.

It must be noted that in the determination of $E[R_k(t)]$, as the terms are summed over i , the whole term was found to be zero when $S_{ik} = 0$. L'Hospital's rule was used for this step.

Combining equations (10) and (20), we get:

$$E[R(t)] = \beta_L \sum_{k=1}^L \bar{\eta} \lambda \bar{A}_k \sum_{i=1}^N \bar{G}_i V_{ik} \bar{\phi}_{ik} w_{ik} \quad (27)$$

in which $\bar{\eta} = 1.0$.

To find the variance of $R(t)$, we use the relation in equation (11), i.e.:

$$\begin{aligned} \text{Var}[R(t)] = & \beta_L \left\{ \sum_{k=1}^L \text{Var}[R_k(t)] \right. \\ & \left. + 2 \sum_{k=1}^L \sum_{m=k+1}^L \text{Cov}[R_k(t), R_m(t)] \right\} \end{aligned}$$

in which

$$\text{Var}[R_k(t)] = E[R_k^2(t)] - E^2[R_k(t)] \quad (28)$$

Equation (28) is solved below. First we obtain the variance of $R_k(t)$ conditional on η ; for a filtered Poisson process:

$$\begin{aligned} \text{Var}[R_k(t) | \eta = \eta_0] &= \lambda \int_0^{t_{\text{DEL}}} E[|w(t, \tau, y)|^2] d\tau \\ &= \lambda \eta_0^2 \int_0^{t_{\text{DEL}}} E[|A_k \sum_{i=1}^N G_i V_{ik} \theta_{ik}(\tau) \phi_{ik}|^2] d\tau \quad (29) \end{aligned}$$

Assuming that A^2 and D^2 are uncorrelated, equation (29) above can be written as:

$$\text{Var}[R_k(t) | \eta] = \lambda \eta_0^2 \int_0^{t_{\text{DEL}}} E[A_k^2] E\left[\left|\sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik}(\tau) \phi_{ik}\right|^2\right] d\tau \quad (30)$$

Define

$$g(D_k) = \sum_{i=1}^N G_i V_{ik} \theta_{ik}(\tau) \phi_{ik} \quad (31)$$

Substituting equation (31) into equation (30), we get:

$$\text{Var}[R_k(t) | \eta] = \lambda \eta_0^2 \int_0^{t_{\text{DEL}}} E[A_k^2] E[g^2(D_k)] d\tau \quad (32)$$

where:

$$E[A_k^2] = \bar{A}_k^2 + \sigma_{A_k}^2$$

Using the Taylor series expansion explained in equations (21) through (24), we can state that:

$$E[g^2(x)] \approx g^2(E[x]) + \left[\frac{\partial g(x)}{\partial x}\right]_x^2 \sigma_x^2 \quad (32)$$

Using the above relation, we can find $E[g^2(D_k)]$:

$$E[g^2(D_k)] \approx \left\{ \sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik}(\tau) \frac{\sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right)}{1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2} \right\}^2 + \left[\sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik}(\tau) \left[\frac{\partial}{\partial D_k} \frac{\sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right)}{1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2} \right] \right] \sigma_{D_k}^2 \quad (33)$$

let

$$B_{ik} = \frac{\partial}{\partial D_k} \left\{ \frac{\sinh\left(\frac{S_{ik} D_k}{2}\right)}{1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2} \right\} \sigma_{D_k}$$

$$B_{ik} = \sigma_{D_k} \left\{ \frac{\left(\frac{S_{ik}}{2}\right) \cosh\left(\frac{S_{ik} \bar{D}_k}{2}\right)}{1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2} - \frac{2 \left(\frac{S_{ik}}{2\pi}\right)^2 \sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right)}{\left[1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2\right]^2} \right\} \quad (34)$$

In the above relations \bar{D}_k is the mean duration of the load in the k th period (which is related to the load frequency: $w = \pi/D$). This mean value and the corresponding variance of load duration may change from one period to another as the speed of moving traffic changes with seasonal weather conditions. An example of this change can be observed as the speeds of cars go down as the snow and rain prevail during the winter season.

Define

$$\bar{\alpha}_{ik} = \frac{\sinh\left(\frac{S_{ik} \bar{D}_k}{2}\right)}{1 + \left(\frac{S_{ik} D_k}{2\pi}\right)^2} \quad (35)$$

Substituting the expressions in equation (34) and (35) into equation (33); and equation (33) into equation (32), we finally get:

$$\begin{aligned} \text{Var}[R_k(t) | \eta] = & \lambda \eta_0^2 (\sigma_{A_k}^2 + \bar{A}_k^2) \left\{ \int_0^{t_{\text{DEL}}} \left(\sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik}(\tau) \right. \right. \\ & \left. \left. \bar{\alpha}_{ik} \right)^2 d\tau + \int_0^{t_{\text{DEL}}} \left(\sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik}(\tau) B_{ik} \right)^2 d\tau \right\} \quad (36) \end{aligned}$$

If we define

$$I_k^1 = \int_0^{t_{\text{DEL}}} \left(\sum_{i=1}^N \bar{G}_i V_{ik} \bar{\alpha}_{ik} \theta_{ik}(\tau) \right)^2 d\tau \quad (37)$$

$$\begin{aligned} I_k^1 = & \sum_{i=1}^N \bar{G}_i^2 V_{ik}^2 \bar{\alpha}_{ik}^2 \int_0^{t_{\text{DEL}}} \theta_{ik}^2(\tau) d\tau \\ & + 2 \sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j V_{ik} V_{jk} \bar{\alpha}_{ik} \bar{\alpha}_{jk} \\ & \times \int_0^{t_{\text{DEL}}} \theta_{ik}(\tau) \theta_{jk}(\tau) d\tau \quad (38) \end{aligned}$$

From which we get:

$$\begin{aligned} I_k^1 = & \sum_{i=1}^N \bar{G}_i^2 V_{ik}^2 \bar{\alpha}_{ik}^2 \left[\frac{1}{2S_{ik}} \{ \exp(2 S_{ik} t_{\text{DEL}}) - 1 \} \right. \\ & \left. + 2 \sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j V_{ik} V_{jk} \bar{\alpha}_{ik} \bar{\alpha}_{jk} \left[\frac{1}{(S_{ik} + S_{jk})} \{ \exp((S_{ik} + S_{jk}) t_{\text{DEL}}) - 1 \} \right] \right] \quad (39) \end{aligned}$$

Similarly, we can write:

$$I_k^2 = \int_0^{t_{\text{DEL}}} \left(\sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik} \bar{B}_{ik} \right)^2 d\tau$$

$$\begin{aligned}
&= \sum_{i=1}^N \bar{G}_i^2 V_{ik}^2 \bar{B}_{ik}^2 \left[\frac{1}{2S_{ik}} \exp(2 S_{ik} t_{DEL}) - 1 \right] \\
&+ 2 \sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j V_{ik} V_{jk} B_{ik} B_{jk} \left[\frac{\exp\{(S_{ik} + S_{jk}) t_{DEL}\}}{(S_{ik} + S_{jk})} \right] \quad (40)
\end{aligned}$$

From equations (39) and (40), equation (36) becomes:

$$\text{Var}[R_k(t) | \eta] = \lambda \eta_o^2 (\sigma_{A_k}^2 + \bar{A}_k^2) (I_k^1 + I_k^2) \quad (41)$$

To compute $\text{Var}[R_k(t)]$, first we write:

$$\text{Var}[R_k(t) | \eta] = E[R_k^2(t) | \eta] - E^2[R_k(t) | \eta] \quad (42)$$

From which we get:

$$E[R_k^2(t) | \eta] = \text{Var}[R_k(t) | \eta] + E^2[R_k(t) | \eta] \quad (43)$$

Both $\text{Var}[R_k(t) | \eta]$ and $E^2[R_k(t) | \eta]$ in the above relation have been computed in equations (41) and (19) respectively.

Therefore:

$$\begin{aligned}
E[R_k^2(t) | \eta] &= \lambda \eta_o^2 (\sigma_{A_k}^2 + \bar{A}_k^2) (I_k^1 + I_k^2) \\
&+ (\lambda \eta_o \bar{A}_k \sum_{i=1}^N \bar{G}_i V_{ik} \bar{\phi}_{ik} w_{ik})^2 \quad (44)
\end{aligned}$$

If we define:

$$I_k^3 = \left(\sum_{i=1}^N \bar{G}_i V_{ik} \bar{\phi}_{ik} w_{ik} \right)^2 \quad (45)$$

$$I_k^3 = \sum_{i=1}^N \bar{G}_i^2 V_{ik}^2 \bar{\phi}_{ik}^2 w_{ik}^2 + 2 \sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j V_{ik} \bar{\phi}_{ik} \phi_{jk} w_{ik} w_{jk} \quad (46)$$

Substitution of equations (45) or (46) into equation (44) yields:

$$E[R_k^2(t) | \eta] = \lambda \eta_0^2 (\sigma_A^2 + \bar{A}^2) (I_k^1 + I_k^2) + \lambda^2 \eta_0^2 \bar{A}^2 I_k^3 \quad (47)$$

Taking the expectation over η , we have

$$E[R_k^2(t)] = E_\eta [R_k^2(t) | \eta] \quad (48)$$

From which we get

$$E[R_k^2(t)] = \lambda E[\eta^2] \{ (\sigma_A^2 + \bar{A}^2) (I_k^1 + I_k^2) + \lambda \bar{A}^2 I_k^3 \} \quad (49)$$

But since $E[\eta^2] = \sigma_\eta^2 + \bar{\eta}^2$ and since $\eta=1.0$, equation (49)

becomes:

$$E[R_k^2(t)] = (\sigma_\eta^2 + 1) \{ (\sigma_A^2 + \bar{A}^2) (I_k^1 + I_k^2) + \lambda \bar{A}^2 I_k^3 \} \quad (50)$$

Since

$$\text{Var}[R_k(t)] = E[R_k^2(t)] - E^2[R_k(t)] \quad (51)$$

which have been obtained in equation (50) and (20) respectively, one can write:

$$\begin{aligned} \text{Var}[R_k(t)] = & \lambda(\sigma_n^2 + 1) \{ (\sigma_A^2 + \bar{A}^2) (I_k^1 + I_k^2) + \lambda \bar{A}_k^2 I_k^3 \} \\ & - \lambda^2 \bar{A}_k^2 I_k^3 \end{aligned} \quad (52)$$

$$\begin{aligned} \text{Var}[R_k(t)] = & \lambda [(\sigma_n^2 + 1) \{ (\sigma_A^2 + \bar{A}^2) (I_k^1 + I_k^2) + \lambda \bar{A}_k^2 I_k^3 \} \\ & - \lambda \bar{A}_k^2 I_k^3] \end{aligned} \quad (53)$$

Referring back to equation (11), we have

$$\begin{aligned} \text{Var}[R(t)] = & (\beta_L^2) \left\{ \sum_{k=1}^L \text{Var}[R_k(t)] \right. \\ & \left. + 2 \sum_{k=1}^L \sum_{m=k+1}^L \text{Cov}[R_k(t), R_m(t)] \right\} \end{aligned}$$

In which $\text{Var}[R_k(t)]$ has been found in equation (53) above, and $\text{Cov}[R_k(t), R_m(t)]$ is yet to be determined. We use the relation in equation (14) for filtered Poisson processes:

$$\text{Cov}[R_k(t), R_m(t)] = \lambda \int_0^t \text{DEL}_E [w_k(t, \tau, y) w_m(t, \tau, y)] d\tau$$

Substituting for $w_k(t, \tau, y)$, $w_m(t, \tau, y)$ as done in equations (15) and (29), we get:

$$\text{Cov}[R_k(t), R_m(t)] = \lambda \int_0^{t_{\text{DEL}}} E\left[\left\{A_k \eta \sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik}(\tau) \phi_{ik}\right\} \left\{A_m \eta \sum_{i=1}^N \bar{G}_i V_{im} \theta_{im}(\tau) \phi_{im}\right\}\right] d\tau \quad (54)$$

Assuming that η doesn't change its statistical characteristics with time, i.e., even with aging in the pavement materials, their statistical distribution will be maintained the same, we get from equation (54):

$$\text{Cov}[R_k(t), R_m(t)] = \lambda E[A_k A_m] E[\eta^2] \int_0^{t_{\text{DEL}}} E\left[\left(\sum_{i=1}^N \bar{G}_i V_{ik} \theta_{ik}(\tau) \phi_{ik}\right) \left(\sum_{i=1}^N \bar{G}_i V_{im} \theta_{im}(\tau) \phi_{im}\right)\right] d\tau \quad (55)$$

Using the above assumption for η to be applied for the load amplitude (i.e., the independence of statistical characteristics of load amplitude from seasonal changes), equation (55) becomes:

$$\text{Cov}[R_k(t), R_m(t)] = \lambda E[A^2] E[\eta^2] \sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j V_{ik} V_{jm} E[\phi_{ik} \phi_{jm}] \int_0^{t_{\text{DEL}}} \theta_{ik}(\tau) \theta_{jm}(\tau) d\tau \quad (56)$$

Define:

$$H_{ijkm} = \int_0^{t_{\text{DEL}}} \theta_{ik}(\tau) \theta_{jm}(\tau) d\tau \quad (57)$$

$$H_{ijkm} = \int_0^{t_{\text{DEL}}} \exp(\tau S_{ik}) \exp(\tau S_{jm}) d\tau$$

$$H_{ijkm} = \frac{\exp[t_{\text{DEL}}(S_{ik} + S_{jm})] - 1}{(S_{ik} + S_{jm})} \quad (58)$$

Also define:

$$s_{ijkm}(D) = \phi_{ik} \phi_{jm} \quad (59)$$

$$E[s_{ijkm}(D)] = E\left[\frac{\sinh\left(\frac{s_{ik} D_k}{2}\right)}{1 + \left(\frac{s_{ik} D_k}{2\pi}\right)^2} \frac{\sinh\left(\frac{s_{jm} D_m}{2}\right)}{1 + \left(\frac{s_{jm} D_m}{2\pi}\right)^2}\right] \quad (60)$$

If we keep the statistical properties of D_k unchanged with time, we get $D_k \equiv D_m \equiv D$. Therefore, equation (60) becomes:

$$E[s_{ijkm}(D)] = E\left[\frac{\sinh\left(\frac{s_{ik} s_{jm} D^2}{4}\right)}{1 + \left\{\left(\frac{s_{ik}}{2\pi}\right)^2 + \left(\frac{s_{jm}}{2\pi}\right)^2\right\} D^2 + \left(\frac{s_{ik} s_{jm}}{4\pi^2}\right)^2 D^4}\right] \quad (61)$$

The second-order approximation for the above expression as given by equation (23) becomes:

$$E[s_{ijkm}(D)] = s_{ijkm}(\bar{D}) + \frac{1}{2} \left[\frac{\partial^2 s_{ijkm}(D)}{\partial D^2} \Big|_{\bar{D}} \right]^2 \sigma_D^2 \quad (62)$$

$$\begin{aligned} \frac{\partial s(D)}{\partial D} &= \frac{\left(\frac{s_{ik} s_{jm}}{4}\right) (2D) \cosh\left(\frac{s_{ik} s_{jm} D^2}{4}\right)}{1 + \left\{\left(\frac{s_{ik}}{2\pi}\right)^2 + \left(\frac{s_{jm}}{2\pi}\right)^2\right\} D^2 + \left(\frac{s_{ik} s_{jm}}{4\pi^2}\right)^2 D^4} \\ &\quad - \frac{\sinh\left(\frac{s_{ik} s_{jm} D^2}{4}\right) \left[2D \left\{\left(\frac{s_{ik}}{2\pi}\right)^2 + \left(\frac{s_{jm}}{2\pi}\right)^2\right\} + 4D^3 \left(\frac{s_{ik} s_{jm}}{4\pi^2}\right)^2\right]}{\left[1 + \left\{\left(\frac{s_{ik}}{2\pi}\right)^2 + \left(\frac{s_{jm}}{2\pi}\right)^2\right\} D^2 + \left(\frac{s_{ik} s_{jm}}{4\pi^2}\right)^2 D^4\right]^2} \end{aligned} \quad (63)$$

If we define:

$$\mu_{ijkm} = \frac{s_{ik} s_{jm}}{2} \quad (64a)$$

$$v_{ik} = \frac{s_{ik}}{2} \quad (64b)$$

and,

$$K_{jm} = \frac{S_{jm}}{2} \quad (64c)$$

The second partial differential of s_{ijkm} evaluated at the mean value of D , becomes:

$$\begin{aligned} \frac{\partial^2 s_{ijkm}(D)}{\partial D^2} \Big|_{\bar{D}} &= \mu_{ijkm} \frac{\cosh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right) + \mu_{ijkm} \bar{D}^2 \sinh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right)}{1 + \left[\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right] \bar{D}^2 + \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2 \bar{D}^4} \\ &- \frac{\mu_{ijkm} \bar{D} \cosh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right) \{2\bar{D} \left[\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right] + 4\bar{D}^3 \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2\}}{\{1 + \left[\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right] \bar{D}^2 + \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2 \bar{D}^4\}^2} \\ &- \frac{\mu_{ijkm} \bar{D} \cosh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right) [2\bar{D} \left\{\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right\} + 4\bar{D}^3 \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2]}{\{1 + \left[\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right] \bar{D}^2 + \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2 \bar{D}^4\}^2} \\ &- \frac{\sinh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right) [2\left\{\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right\} + 12\bar{D}^2 \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2]}{\{1 + \left[\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right] \bar{D}^2 + \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2 \bar{D}^4\}^2} \\ &+ \frac{2 \sinh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right) \{2\bar{D} \left[\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right] + 4\bar{D}^3 \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2\}}{\{1 + \left[\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2\right] \bar{D}^2 + \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2 \bar{D}^4\}^3} \quad (65) \end{aligned}$$

Let:

$$\phi_{mk} = \frac{1}{1 + \left[\left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{mj}}{\pi}\right)^2\right] \bar{D}^2 + \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2 \bar{D}^4} \quad (66a)$$

$$J_{mk}^1 = \cosh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right) \quad (66b)$$

$$J_{mk}^2 = \sinh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right) \quad (66c)$$

$$\Gamma_{mk} = \left(\frac{v_{ik}}{\pi}\right)^2 + \left(\frac{K_{jm}}{\pi}\right)^2 \quad (66d)$$

Substituting for the above relations in equation (65), we get:

$$\begin{aligned} \frac{\partial^2 s_{ijkm}(D)}{\partial D^2} \Big|_{\bar{D}} &= \mu_{ijkm} (J_{mk}^1 + \mu_{ijkm} \bar{D}^2 J_{mk}^2) \phi_{mk} \\ &\quad - 4 \mu_{ijkm} \bar{D} J_{mk}^1 \{ \bar{D} \Gamma_{mk} + 2 \bar{D}^3 \left(\frac{\mu_{ijkm}}{2}\right)^2 \} (\phi_{mk})^2 \\ &\quad - 2 J_{mk}^2 \{ \Gamma_{mk} + 6 \bar{D}^2 \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2 \} (\phi_{mk})^2 \\ &\quad - 4 J_{mk}^2 \{ \Gamma_{mk} \bar{D} + 2 \bar{D}^3 \left(\frac{\mu_{ijkm}}{2\pi^2}\right)^2 \} (\phi_{mk})^3 \quad (67) \end{aligned}$$

If we let

$$s''(\bar{D}) = \frac{\partial^2 s_{ijkm}(D)}{\partial D^2} \Big|_{\bar{D}} \quad (68)$$

Equation (62) above may be written as:

$$E[s_{ijkm}(D)] = \sinh\left(\frac{\mu_{ijkm} \bar{D}^2}{2}\right) \phi_{mk} + \frac{1}{2} s''(\bar{D}) \sigma_D^2 \quad (69)$$

Substituting equations (69) and (58) back into equation (56), we get the expression for the Covariance as follows:

$$\text{Cov}[R_k(t), R_m(t)] = (\sigma_A^2 + \bar{A}^2)(\sigma_\eta^2 + 1)$$

$$\sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j V_{ik} V_{jm} \left\{ \text{Sinh} \left(\frac{\mu_{ijkm} \bar{D}^2}{2} \right) \phi_{mk} + \frac{1}{2} \zeta''(\bar{D}) \sigma_D^2 \right\} H_{mk}$$

Equation (76) is substituted back into equation (11) to get:

$$\text{Var}[R(t)] (\beta_L^2) \left\{ \sum_{k=1}^L \text{Var}[R_k(t)] + 2 \sum_{k=1}^L \sum_{m=k+1}^L \text{Cov}[R_k(t), R_m(t)] \right\}$$

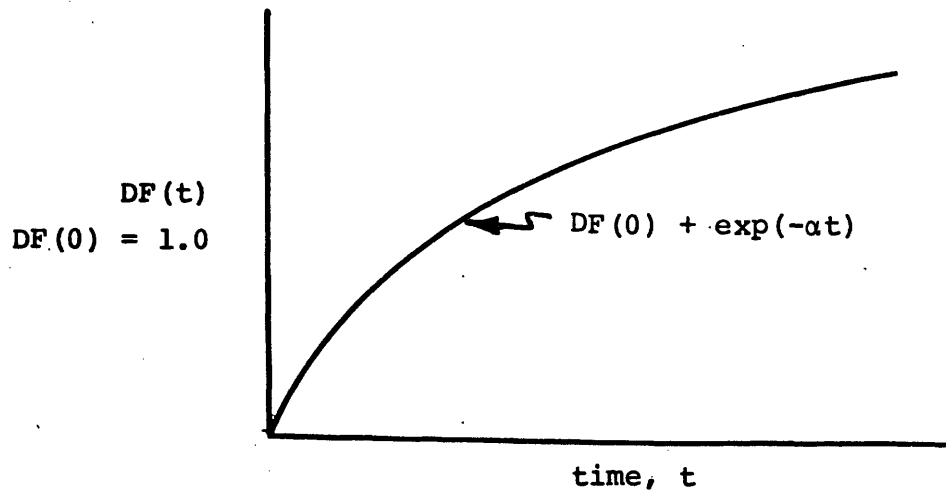
Roughness or Slope Variance

To obtain some measured of roughness, we have to get the spatial correlation of the system. This can be expressed as a function of the spatial correlation coefficient of the materials. From this we obtain the auto-correlation function of the surface deformation. It is obvious here that the solution is based on the assumption that the surface roughness is a function of the spatial variations in the properties of the materials.

It is recognized that this assumption accounts only for the initial stages of development of surface roughness, and neglects all dynamic and internal effects that result from impact forces of moving traffic when such roughness exist in the road. However, a simplifying assumption can be made such that the rate of development of surface roughness will not be under evaluated. Several conceptual assumptions may be suggested. One logical assumption will be the inclusion of a pseudo dynamic factor in the load amplitude. This factor can be either continuous or discrete in time. This factor will be multiplied by the load amplitude, and is increased, exponentially for instance, with time. If we call this factor D.F., for dynamic factor, we can have the following relation.

$$DF(t) = DF(0) + e^{-\alpha t} \quad (71)$$

Where $DF(0)$ is the initial time dynamic factor, and for all practical purposes can be set to 1.0. The growth of the dynamic factor with time is shown in Figure :



The term α is a material characteristic and is generally higher for stiffer materials and lower for softer ones.

It is of interest to present a supporting argument for the fact that roughness develops because of the spatial variation in the properties of the materials in the different layers. Let us imagine a highway pavement built from a 5 ft. thick steel panel over which the traffic passes in a manner similar to highway pavements. One would expect that the surface of the panel will not deflect considerably. However if the same pavement is made of a 1"-thick of steel plate over a compacted soil layer, a wavy surface is expected after a few passes of traffic loads. In the first case the steel panel is uniform, or at least much more uniform than the synthetic materials made from asphalt or concrete. The effect

of the variation in the soil beneath the panel is considerably reduced if not almost eliminated by the thickness of the panel. However, in the second case even though the surface plate is highly uniform, the material beneath is highly inhomogeneous and its effect on the deflection of the plate is transmitted through the relatively low thickness of the plate. The extension of this argument through logical deduction to the case of highway pavements of different layers of varying homogeneities is not difficult to visualize.

The analysis below has been used to obtain the spatial auto correlation function of the surface deflection. From which, the slope variance is fairly easily obtained. The spatial auto correlation function of a system's response $\tilde{R}_t(x)$ can be expressed as:

$$\tilde{R}_t(x) = E_x[R(t, x_0) R(t, x_\ell)] - E[R(t, x_0)] E[R(t, x_\ell)] \quad (72)$$

where: $E_x[]$ signifies the expectation is taken over space only, and $R(t, x_0)$ is the response of the system at time t at a location (0) . In this case the response is the vertical deflection at the surface of the pavement. Equation (72) above can be expressed as follows:

$$\begin{aligned} \tilde{R}_t(x) = & E_x \left[\left\{ \sum_{k=1}^L R_k(t, x_0) \right\} \left\{ \sum_{k=1}^L R_k(t, x_\ell) \right\} \right] \\ & - E_x[R(t, x_0)] E_x[R(t, x_\ell)] \end{aligned} \quad (73)$$

Since the roughness of the road, at least initially is totally attributed to spatial variation in the materials' properties, the only space variables in the above expressions will be η , which in this case can be written as η_0 and η_ℓ to be related to points (x_p) and (x_ℓ) respectively.

In equation (73) above, we have

$$R_k(t, x_0) = \eta_{x_0} \sum_{j=1}^{\eta_k} A_j \sum_{i=1}^N G_i V_{ik} \theta_{ik}(\tau_j) \phi_{ik} \quad (74)$$

If we call

$$Z_k = \sum_{j=1}^{\eta_k} A_j \sum_{i=1}^N G_i V_{ik} \theta_{ik}(\tau_j) \phi_{ik}$$

then equation (73) above will become:

$$\begin{aligned} \tilde{R}_t(x) &= E_x[\eta_{x_0} Z_k \cdot \eta_{x_\ell} Z_k] - E_x[R(t, x_0)] E_x[R(t, x_\ell)] \\ &= E[\eta_{x_0} \eta_{x_\ell}] Z_k^2 - E[R(t, x_0)] E[R(t, x_\ell)] \end{aligned} \quad (75)$$

In the above equation, we have:

$$E[\eta_{x_0}, \eta_{x_\ell}] = \text{Cov}[\eta_{x_0}, \eta_{x_\ell}] + E[\eta_{x_0}] E[\eta_{x_\ell}] \quad (76)$$

In which

$$E[\eta_{x_0}] = E[\eta_{x_\ell}] = 1.0 \quad (77)$$

and:

$$\text{Cov}[\eta_{x_0} \eta_{x_l}] = \rho_{x_0 x_l} \sigma_{\eta_{x_0}} \sigma_{\eta_{x_l}} \quad (78)$$

Assuming that the process (spatial correlation of materials properties) is a homoscedastic process*, equation (8) above becomes:

$$\text{Cov}[\eta_{x_0} \eta_{x_l}] = \rho_{x_0 x_l} \sigma_{\eta}^2 \quad (78a)$$

Substituting equations (77) and (78a) into equation (75) yields:

$$\begin{aligned} \tilde{R}_t(x) = & [\rho_{x_0 x_l} \sigma_{\eta}^2 + 1] \sum_{k=1}^L z_k^2 \\ & - E[R(t, x_0)] E[R(t, x_l)] \end{aligned} \quad (79)$$

$\rho_{x_0 x_l}$ is the spatial correlation coefficient for the surface deflection which is related to the properties of the materials by solving for the step response of the system.

$\rho_{x_0 x_l}$ can be measured, and different studies in the area of structures have revealed that different exponential functions can be used to represent the spatial correlation behavior. A survey of different models has been done in the area of structural floor loading in England [H5] and

elsewhere. A model which seems to fit best the pavement

* A homoscedastic process is one in which the variance doesn't have spatial or temporal variation. It is to be distinguished from heteroscedastic variance processes.

systems can be of the form:

$$\rho_{\eta_{x_0} \eta_{x_l}} = A + B e^{-|x_0 - x_l|^2 / C^2} \quad (80)$$

where:

$|x_0 - x_l|$ is the absolute distance between the two points. A is the minimum correlation between points spanning very far apart from each other.

B and C are material's variables. It should be noted that $A + B = 1$ and that there is a special case of this where $A = 0$, i.e.,

$$\rho_{\eta_{x_0} \eta_{x_l}} = e^{-|x_0 - x_l|^2 / C^2} \quad (81)$$

If we replace $x = |x_0 - x_l|$ we get

$$\rho_{x_0 x_l} = A + B e^{-|x_l|^2 / C^2} \quad (82)$$

If we substitute for the value of the spatial correlation coefficient $\rho_{x_0 x_l}$ (in equation 82) into equation (79), the resulting spatial auto correlation function will be:

$$\begin{aligned} \tilde{R}_t(x) = & [(A + B e^{-|x|^2 / C^2}) \sigma_{\eta}^2 + 1] \sum_{k=1}^L z_k^2 \\ & - E[R(t, x_0)] E[R(t, x_l)] \end{aligned} \quad (83)$$

If we define $Z(t)$ as the first space derivative of a function $Z(t)$, and if $S(t)$ represents the surface deformation of a body, then $\dot{S}(t)$ will be the slope of the surface as a function of time.

Since:*

$$\sigma_{\dot{S}(t)}^2 = - \left. \frac{\partial^2 R_t(x)}{\partial x^2} \right|_{|x|=0} \quad (84)$$

where:

$$\left. \frac{\partial^2 R_t(x)}{\partial x^2} \right|_{|x|=0}$$

is the second derivative of the auto correlation function evaluated at $|x| = 0$. And $\sigma_{\dot{S}(t)}^2$ is the slope variance (identical to that used by AASHO in its serviceability measures of roughness). The second term (i.e., $E[R(t, x_0)]$ $E[R(t, x_l)]$ is not a function of x , therefore:

$$\begin{aligned} \left. \frac{\partial^2 R_t(x)}{\partial x^2} \right|_{|x|=0} &= \frac{2B}{C^2} e^{-\left(\frac{x^2}{C^2}\right)} \left[\frac{2x^2}{C^2} - 1 \right] \bigg|_{x=0} \left(\sum_{k=1}^L z_k \right)^2 \\ &= - \frac{2B}{C^2} \left(\sum_{k=1}^L z_k \right)^2 \end{aligned} \quad (85)$$

From which:

$$\sigma_{\dot{S}(t)}^2 = \frac{2B}{C^2} \left(\sum_{k=1}^L z_k \right)^2 \quad (86)$$

* Proof of this formula can be found in Reference [F2]

This relation is used in the serviceability equation whether in the linearized model proposed below or in the AASHO regression equation.

Since the term $\sigma_S^2(t)$ is a random variable due to the randomness in loads histories (duration, amplitude, and number), one can find the first and second moments of the slope variance in the following manner.

$$\begin{aligned} E[\sigma_S^2(t)] &= \frac{2B}{C^2} E\left[\sum_{k=1}^L z_k\right]^2 \\ &= \frac{2B}{C^2} \left\{ \sum_{k=1}^L E[z_k^2] + 2 \sum_{k=1}^L \sum_{m=k+1}^L \text{Cov}[z_k, z_m] \right\} \quad (87) \end{aligned}$$

If η was omitted from equation (47), we would have:

$$E[z_k^2] = E[R_k^2(t) | \eta] \quad (88)$$

Therefore if we rewrite equation (47) without η , we get:

$$E[R_k^2(t) | \eta] = \lambda(\sigma_{A_k}^2 + \bar{A}_k^2)(I_k^1 + I_k^2) + \lambda^2 \bar{A}_k^2 I_k^3 \quad (89)$$

Also from equation (70), omitting the effect of η , we have:

$$\begin{aligned} \text{Cov}[z_k(t), z_m(t)] &= \lambda(\sigma_A^2 + \bar{A}^2) \sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j V_{ik} V_{jm} \\ &\quad \left\{ \sinh\left(\frac{\mu_{ijkm}}{2} \bar{D}^2\right) \phi_{mk} + \frac{1}{2} s''(D) \sigma_D^2 \right\} H_{mk} \quad (90) \end{aligned}$$

Therefore:

$$E[\sigma_S^2(t)] = \frac{2B}{C^2} (\beta^2) \left\{ \left[\sum_{k=1}^L \lambda (\sigma_A^2 + \bar{A}^2) (I_k^1 + I_k^2) + \lambda^2 \bar{A}^2 I_k^3 \right] + 2 \sum_{k=1}^L \sum_{m=k+1}^L \text{Cov}[Z_k, Z_m] \right\} \quad (91)$$

The variance of the slope variance is a little more difficult to obtain. First consider the following:

$$\text{Var}[\sigma_S^2(t)] = \left[\frac{2B}{C^2} \right]^2 (\beta_L^4) \text{Var} \left[\sum_{k=1}^L Z_k^2(t) \right] \quad (92)$$

If we assume that Z_k^2 are uncorrelated, we get:

$$\text{Var}[\sigma_S^2(t)] = \left[\frac{2B}{C^2} \right]^2 (\beta_L^4) \sum_{k=1}^L \text{Var}[Z_k^2(t)] \quad (93)$$

An approximating assumption will be used here by involving the central limit theorem. Since $Z(t)$ is a random process resulting from a sum of random variables, then one can assume that $Z(t)$ has a Gaussian distribution. This assumption is made to make use of already existing solutions for higher order moments of Gaussian processes. Benjamin and Cornell [B1] have integrated the probability density function of a Gaussian distribution to obtain the higher central moments. The "even-order" higher moments are given by

$$\begin{aligned} \mu_X^{(n)} &= E[(X - m_X)^n] \\ &= \frac{n!}{2^{n/2} (\frac{n}{2})!}, \quad n = 2, 4, 6, \dots \end{aligned} \quad (94)$$

This expression has been expanded for this analysis to obtain the fourth-order moment, as follows:

$$\begin{aligned}
 \mu_X^{(4)} &= E[(X-m_X)^4] \\
 &= E[X^4 - 4m_X X^3 + 6m_X^2 X^2 - 4m_X^3 X + m_X^4] \\
 \mu_X^{(4)} &= E[X^4] - 4m_X E[X^3] + 6m_X^2 E[X^2] - 4m_X^3 E[X] + m_X^4
 \end{aligned} \tag{95}$$

To evaluate $E[X^3]$, let us first evaluate $E[(X-m_X)^3]$. Since the normal distribution is symmetrical, then all odd-order moments are zero. Therefore:

$$\mu_X^{(3)} = E[(X-m_X)^3] \tag{96}$$

Equation (96) yields:

$$E[X^3 - 3m_X X^2 + 3m_X^2 X - m_X^3] = 0 \tag{97}$$

From which we get:

$$E[X^3] = m_X [3 \sigma_X^2 + m_X^2] \tag{98}$$

Substituting equation (98) into equation (95) yields

$$\begin{aligned}
 \mu_X^{(4)} &= E[X^4] - 4m_X^2 [3 \sigma_X^2 + m_X^2] + 6m_X^2 (m_X^2 + \sigma_X^2) + m_X^4
 \end{aligned} \tag{99}$$

Substituting for the value of $\mu_X^{(4)}$ in equation (94)

$$\mu_X^{(4)} = \frac{4!}{2^2 (2)!} = 3 \sigma_X^4 \quad (100)$$

Combining equation (99) and (100) yields:

$$E[X^4] = m_X^4 + 6m_X^2 \sigma_X^2 + 3 \sigma_X^4 \quad (101)$$

Going back to equation (93), we have:

$$\text{Var}[Z_k^2(t)] = E[Z_k^4(t)] - E^2[Z_k^2(t)] \quad (102)$$

where

$$\begin{aligned} E[Z_k^4(t)] &= E^4[Z_k(t)] + 6E^2[Z_k(t)] \text{Var}[Z_k(t)] \\ &\quad + 3 \text{Var}^2[Z_k(t)] \end{aligned} \quad (103)$$

where

$$E[Z_k(t)] = E[R_k(t) | \eta] \text{ for } \eta = 1 \quad (104)$$

and

$$\text{Var}[Z_k(t)] = \text{Var}[R_k(t) | \eta] \text{ for } \eta = 1 \quad (105)$$

which are obtained in equation (20) and (41) respectively and $E^2[Z_k^2]$ is found from equation (89) above.

Substituting equation (102) into equation (93) we get the value of $\text{Var}[\sigma_S^2(t)]$

Probabilistic Formulation of Miner's Law

$$D(t) = \sum_{k=1}^L \frac{n_k}{N_k} \quad (106)$$

$$E[D(t)] = E\left[\sum_{k=1}^L \frac{n_k}{N_k}\right] \quad (107)$$

$$E[D(t)] = \sum_{k=1}^L E\left(\frac{n_k}{N_k}\right) \quad (107a)$$

To obtain this value, a multidimensional Taylor series expansion is required. The second order approximation to the expected value of $D(t)$ is:

$$E[D(t)] = \sum_{k=1}^L \frac{E[n_k]}{E[N_k]} + \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^2 \left. \frac{d^2(D(t))}{dn_k dn_k} \right|_m \text{Cov}[n_k, N_k] \quad (108)$$

The term $\left. \frac{d^2(D(t))}{dn_k dn_k} \right|_m$ is a mixed second partial derivative of $D(t)$ with respect to n_k and N_k evaluated at $E[n_k]$, $E[N_k]$. However, since n_k and N_k are not correlated, equation (108) above reduces to:

$$E[D(t)] = \sum_{k=1}^L \frac{E[n_k]}{E[N_k]} + \frac{1}{2} \left[\left. \frac{\partial^2 D(t)}{\partial n_k^2} \right|_m \sigma_{n_k}^2 + \left. \frac{\partial^2 D(t)}{\partial N_k^2} \right|_m \sigma_{N_k}^2 \right] \quad (109)$$

But

$$\frac{\partial^2 D(t)}{\partial n_k^2} = 0 \quad (110)$$

$$\left. \frac{\partial^2 D(t)}{\partial N_k^2} \right|_m = \frac{2n_k}{\bar{N}_k^3} \quad (111)$$

Therefore:

$$E[D(t)] = \sum_{k=1}^L \left\{ \frac{[\bar{n}_k]}{[\bar{N}_k]} + \frac{[\bar{n}_k]}{[\bar{N}_k]^3} \sigma_{N_k}^2 \right\} \quad (112)$$

In the same manner, one can show that:

$$\text{Var}[D(t)] = \left(\frac{\partial D(t)}{\partial n_k} \right)^2 \sigma_{n_k}^2 + \left(\frac{\partial D(t)}{\partial N_k} \right)^2 \sigma_{N_k}^2 \quad (113)$$

$$\text{Var}[D(t)] = \sum_{k=1}^L \left\{ \frac{\sigma_{n_k}^2}{\bar{N}_k^2} + \left(\frac{\bar{n}_k}{\bar{N}_k^2} \right)^2 \sigma_{N_k}^2 \right\} \quad (113a)$$

Since n_k and N_k are not correlated, $E[n_k]$ and $\sigma_{n_k}^2$ are found from the average rate of traffic loads occurring in a Poisson fashion.

$E[N_k]$ and $\text{Var}[N_k]$ can be found from the fatigue relation:

$$N_k = C \left(\frac{1}{\Delta \epsilon_k} \right)^a \quad (114)$$

where $\Delta \epsilon_k$ is the strain amplitude found in the response formulation with previous section. C and a are materials properties.

Again, using the approximation for a multivariate function, and assuming that only a and c are correlated, we get the second order approximation for the expected value:

$$\begin{aligned}\bar{N}_k = E[N_k] &\approx \bar{C} \left(\frac{1}{\Delta \epsilon_k} \right) \bar{a} + \frac{1}{2} \left[\frac{\partial^2 N_k}{\partial C^2} \right] \bigg|_m \sigma_C^2 \\ &+ 2 \frac{\partial^2 N_k}{\partial C \partial a} \bigg|_m \text{Cov}[C, a] + \frac{\partial^2 N_k}{\partial a^2} \bigg|_m \sigma_a^2 \\ &+ \frac{1}{2} \frac{\partial^2 N_k}{\partial (\Delta \epsilon_k)^2} \sigma_{\Delta \epsilon_k}^2\end{aligned}\quad (115)$$

where:

$$\frac{\partial^2 N_k}{\partial C^2} \bigg|_m = 0 \quad (116)$$

$$\frac{\partial^2 N_k}{\partial C \partial a} \bigg|_m = \left(\frac{1}{\Delta \epsilon_k} \right) \bar{a} \ln \left(\frac{1}{\Delta \epsilon_k} \right) \quad (117)$$

$$\frac{\partial^2 N_k}{\partial a^2} \bigg|_m = \bar{C} \left[\ln \left(\frac{1}{\Delta \epsilon_k} \right) \right]^2 \left(\frac{1}{\Delta \epsilon_k} \right) \bar{a} \quad (118)$$

$$\text{Cov}[c, a] = \rho_{c, a} \sigma_C \sigma_a \quad (119)$$

where $\rho_{c, a}$ is the correlation coefficient of C and a, and σ_C, σ_a are the standard deviations of C and a respectively.

$$\frac{\partial^2 N_k}{\partial (\Delta \epsilon_k)^2} = \bar{C} \bar{a} (\bar{a} + 1) \left(\frac{1}{\Delta \epsilon_k}\right)^{\bar{a} + 2} \quad (120)$$

Therefore:

$$\begin{aligned} E[N_k] &= \bar{C} \left(\frac{1}{\Delta \epsilon_k}\right)^{\bar{a}} + [\ln(\frac{1}{\Delta \epsilon_k})] \left(\frac{1}{\Delta \epsilon_k}\right)^{\bar{a}} \rho_{c,a} \sigma_c \sigma_a \\ &\quad + \frac{1}{2} \bar{C} [\ln(\frac{1}{\Delta \epsilon_k})]^2 \left(\frac{1}{\Delta \epsilon_k}\right)^{\bar{a}} \sigma_a^2 \\ &\quad + \frac{1}{2} \bar{C} \bar{a} (\bar{a} + 1) \left(\frac{1}{\Delta \epsilon_k}\right)^{\bar{a} + 2} \sigma_{\Delta \epsilon}^2 \\ &= \left(\frac{1}{\Delta \epsilon_k}\right)^{\bar{a}} \{ \bar{C} + \rho_{c,a} \sigma_c \sigma_a \ln(\frac{1}{\Delta \epsilon_k}) \\ &\quad + \frac{1}{2} \bar{C} \sigma_a^2 [\ln(\frac{1}{\Delta \epsilon_k})]^2 + \frac{1}{2} \bar{C} \bar{a} (\bar{a} + 1) \left(\frac{1}{\Delta \epsilon_k}\right)^2 \sigma_{\Delta \epsilon_k}^2 \} \end{aligned} \quad (121)$$

The first order approximation for the variance is given by:

$$\begin{aligned} \text{Var}[N_k] &= \left(\frac{\partial N_k}{\partial C} \Big|_m \right)^2 \sigma_C^2 + \left(\frac{\partial N_k}{\partial a} \Big|_m \right)^2 \sigma_a^2 \\ &\quad + \left(\frac{\partial N_k}{\partial \Delta \epsilon_k} \Big|_m \right)^2 \sigma_{\Delta \epsilon_k}^2 + 2 \frac{\partial N_k}{\partial C} \Big|_m \frac{\partial N_k}{\partial a} \Big|_m \\ &\quad \times \text{Cov}[C, a] \end{aligned} \quad (122)$$

where

$$\frac{\partial N_k}{\partial C} \Big|_m = \left(\frac{1}{\Delta \epsilon_k}\right)^{\bar{a}} \quad (123)$$

$$\left. \frac{\partial N_k}{\partial a} \right|_m = \bar{C} \left[\ln \left(\frac{1}{\Delta \epsilon_k} \right) \right] \left(\frac{1}{\Delta \epsilon_k} \right)^{\bar{a}} \quad (124)$$

$$\left. \frac{\partial N_k}{\partial (\Delta \epsilon_k)} \right|_m = - \bar{a} \bar{C} \left(\frac{1}{\Delta \epsilon_k} \right)^{\bar{a} + 1} \quad (125)$$

$$\begin{aligned} \text{Var}[N_k] = & \left(\frac{1}{\Delta \epsilon_k} \right)^{2\bar{a}} \sigma_C^2 + \bar{C}^2 \left[\ln \left(\frac{1}{\Delta \epsilon_k} \right) \right]^2 \left(\frac{1}{\Delta \epsilon_k} \right)^{2\bar{a}} \sigma_a^2 \\ & + \bar{a}^2 \bar{C}^2 \left(\frac{1}{\Delta \epsilon_k} \right)^{2\bar{a}} \sigma_{\Delta \epsilon_k}^2 + 2\bar{C} \left(\frac{1}{\Delta \epsilon_k} \right)^{2\bar{a}} \ln \left(\frac{1}{\Delta \epsilon_k} \right) \\ & \times \rho_{C,a} \sigma_C \sigma_a \end{aligned} \quad (126)$$

Substituting equations (121) and (126) into equation (112) we can obtain the expected value and variance for the damage function $D(t)$.

Strain Amplitude for a Varying Load

The strain amplitude can be expressed as

$$\Delta \epsilon_M = \frac{1}{4} A \eta \sum_{i=1}^N \frac{\bar{G}_i \left(1 + e^{-\frac{\gamma \delta_i D}{2}} \right)}{\left[1 + \left(\frac{\delta_i \gamma D}{2\pi} \right)^2 \right]} \quad (127)$$

$$\Delta \epsilon_M = f(A, \eta, D)$$

Using the Taylor series expansion for a multivariate function, the second-order approximation for the expected value and the first-order approximation for the variance of $\Delta \epsilon_M$ are given as follows; assuming that η , D , and A are uncorrelated:

$$E[\Delta\epsilon_M] \approx f(\bar{A}, \bar{\eta}, \bar{D})$$

$$+ \frac{1}{2} \left[\left. \frac{\partial^2 f}{\partial A^2} \right|_m \sigma_A^2 + \left. \frac{\partial^2 f}{\partial \eta^2} \right|_m \sigma_\eta^2 + \left. \frac{\partial^2 f}{\partial D^2} \right|_m \sigma_D^2 \right] \quad (128)$$

$$\text{Var}[\Delta\epsilon_M] \approx \left(\left. \frac{\partial f}{\partial A} \right|_m \right)^2 \sigma_A^2 + \left(\left. \frac{\partial f}{\partial \eta} \right|_m \right)^2 \sigma_\eta^2 + \left(\left. \frac{\partial f}{\partial D} \right|_m \right)^2 \sigma_D^2 \quad (129)$$

where m signifies that the derivatives are evaluated at $\bar{A}, \bar{\eta}$, and \bar{D} .

$$\begin{aligned} E[\Delta\epsilon_M] &= \frac{1}{4} \bar{A} \sum_{i=1}^N \frac{\bar{G}_i (1 + e^{-\frac{\gamma \delta_i \bar{D}}{2}})}{1 + \left(\frac{\gamma \delta_i \bar{D}}{2\pi} \right)^2} \\ &+ \frac{1}{8} \bar{A} \sigma_D^2 \sum_{i=1}^N \frac{\bar{G}_i \left(\frac{\gamma \delta_i}{2} \right)^2 e^{-\frac{\gamma \delta_i \bar{D}}{2}} \left(1 + \frac{\gamma \delta_i}{\pi} \right)}{1 + \left(\frac{\gamma \delta_i \bar{D}}{2\pi} \right)^2} \\ &- \frac{2}{\pi^2} \left[\frac{1 + \left(\frac{\gamma \delta_i \bar{D}}{2} \right) e^{-\frac{\gamma \delta_i \bar{D}}{2}}}{\left(\frac{\gamma \delta_i \bar{D}}{2} \right)^2} \right] \\ &+ \left(\frac{\gamma \delta_i \bar{D}}{2\pi^2} \right)^2 \frac{(1 + e^{-\frac{\gamma \delta_i \bar{D}}{2}})}{\left[1 + \left(\frac{\gamma \delta_i \bar{D}}{2\pi} \right)^2 \right]^3} \end{aligned} \quad (130)$$

The variance is:

$$\begin{aligned} \text{Var}[\Delta\epsilon_M] &\approx \left(\frac{1}{4} \sum_{i=1}^N \frac{\bar{G}_i (1 + e^{-\frac{\gamma \delta_i \bar{D}}{2}})}{1 + \left(\frac{\gamma \delta_i \bar{D}}{2\pi} \right)^2} \right)^2 \sigma_A^2 \\ &+ \left(\frac{1}{4} \bar{A} \sum_{i=1}^N \frac{\bar{G}_i (1 + e^{-\frac{\gamma \delta_i \bar{D}}{2}})}{1 + \left(\frac{\gamma \delta_i \bar{D}}{2\pi} \right)^2} \right)^2 \sigma_\eta^2 \end{aligned}$$

$$(131) \quad + \sigma_D^2 \left[\frac{1}{4} \bar{A} \sum_{i=1}^N \bar{G}_i \frac{-\left(\frac{\gamma \delta_i}{2}\right) e^{-\frac{\gamma \delta_i \bar{D}}{2}} - \frac{\gamma \delta_i \bar{D}}{2} - \frac{2(1+e^{-\frac{\gamma \delta_i \bar{D}}{2}}) \left(\frac{\gamma \delta_i}{2}\right)^2 \bar{D}}{1 + \left(\frac{\gamma \delta_i \bar{D}}{2}\right)^2} \right] 2$$

$$\text{Let: } \alpha_i = 1 + e^{-\gamma \delta_i \bar{D}}$$

$$\beta_i = 1 + \left(\frac{\gamma \delta_i \bar{D}}{2}\right)^2$$

$$\text{and } \theta_i = \frac{\frac{\gamma \delta_i}{2} e^{-\frac{\gamma \delta_i \bar{D}}{2}} - \frac{\gamma \delta_i \bar{D}}{2}}{\beta_i} - \frac{2 \alpha_i \left(\frac{\gamma \delta_i}{2}\right)^2 \bar{D}}{\beta_i^2} \quad (132)$$

$$\text{Var}[\Delta \epsilon_M] = \frac{1}{16} (\sigma_A^2 + \sigma_n^2 \bar{A}^2) \quad (133)$$

$$\begin{aligned} & \left\{ \sum_{i=1}^N \frac{\bar{G}_i \alpha_i^2}{\beta_i} + 2 \sum_{i=1}^N \sum_{j=i+1}^N \frac{\bar{G}_i \bar{G}_j \alpha_i \alpha_j}{\beta_i \beta_j} \right\} \\ & + \frac{1}{16} \sigma_D^2 \bar{A}^2 \left\{ \sum_{i=1}^N (\bar{G}_i \theta_i)^2 + 2 \sum_{i=1}^N \sum_{j=i+1}^N \bar{G}_i \bar{G}_j \theta_i \theta_j \right\} \end{aligned}$$

Serviceability

$$p = C_1 + C_2 \log(1 + \bar{S}\bar{V}) + C_3 \sqrt{C+p} + C_4 \bar{R}\bar{D}^2 \quad (134)$$

where p = the serviceability index

$$D = \sqrt{C+p} \quad (135)$$

$$\begin{aligned} E[P] &= C_1 + C_2 E[\log(1 + \bar{S}\bar{V})] + C_3 E[D^{1/2}] \\ &+ C_4 E[\bar{R}\bar{D}^2] \end{aligned} \quad (136)$$

$$\begin{aligned}
E[\log(1+SV)] &= \log(1+\overline{SV}) + \frac{1}{2} \frac{d^2}{d \overline{SV}^2} [\log(1+SV)]_{SV} \sigma_{SV}^2 \\
&= \log(1+\overline{SV}) - \frac{1}{2} (1+\overline{SV})^2 \sigma_{SV}^2
\end{aligned} \tag{137}$$

$$E[D^{1/2}] = \overline{D}^{1/2} - \frac{1}{8} \overline{D}^{-3/2} \sigma_D^2 \tag{138}$$

$$E[RD^2] = \text{Var}[RD] + \overline{RD}^2 \tag{139}$$

$$\begin{aligned}
E[P] &= C_1 + C_2 \left\{ \log(1+\overline{SV}) - \frac{1}{2} (1+\overline{SV})^2 \sigma_{SV}^2 \right\} + C_3 \left\{ \overline{D}^{-3/2} \sigma_D^2 \right\} \\
&\quad + C_4 \{ \sigma_{RD}^2 + \overline{RD}^2 \}
\end{aligned} \tag{140}$$

$$\begin{aligned}
\text{Var}[P] &= C_2^2 \text{Var}[\log(1+SV)] + C_3^2 \text{Var}[D^{1/2}] \\
&\quad + C_4^2 \text{Var}[RD^2]
\end{aligned} \tag{141}$$

$$\text{Var}[\log(1+SV)] = \left(\frac{1}{1+\overline{SV}} \right)^2 \sigma_{SV}^2 \tag{142}$$

$$\text{Var}[D^{1/2}] = \left(\frac{1}{2} \overline{D}^{-1/2} \right)^2 \sigma_D^2 \tag{143}$$

$$\text{Var}[RD^2] = (2 \overline{RD})^2 \sigma_{RD}^2 \tag{144}$$

$$\text{Var}[P] = C_2^2 \left(\frac{1}{1+\overline{SV}} \right)^2 \sigma_{SV}^2 + \frac{1}{4} \overline{D}^{-1} \sigma_D^2 + 4 \overline{RD}^2 \sigma_{RD}^2 \tag{145}$$

APPENDIX II. LISTING OF THE COMPUTER PROGRAMS

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING

MATERIALS RESEARCH LABORATORY

A PROBABILISTIC EVALUATION MODEL FOR PAVEMENT SYSTEMS

THIS IS THE MAIN PROGRAM FOR THE PROBABILISTIC CLOSED-FORM SOLUTION OF THE RESPONSE OF PAVEMENT SYSTEMS TO A RANDOM SET OF REPEATED LOADS APPLIED IN A POISSON FASHION, I.E., RANDOM INDEPENDENT ARRIVAL WITH EXPONENTIAL DISTRIBUTION OF THE TIME BETWEEN ARRIVALS. THE TEMPERATURES VARY FROM ONE PERIOD TO ANOTHER ACCORDING TO THE REGIONAL CLIMATIC CONDITIONS REPRESENTED. THE INPUTS TO THIS PROGRAM ARE OF THREE CATEGORIES: LOAD ASSOCIATED INPUTS, MATERIALS VARIABLES, AND TEMPERATURE TEMPLATES FOR THE DESIRED REGION. THE PROGRAM ACTS AS AN EXECUTIVE UNIT CALLING ON THE DIFFERENT SUBROUTINES AND HANDLING THE INTERMEDIATE INFORMATION FLOW. IT ALSO HANDLES THE INPUT/OUTPUT PROCESSES AND ACTS AS A CENTRAL COMMAND UNIT IN THIS CAPACITY.

THE FOLLOWING PARAMETERS CONSTITUTE THE INPUTS TO THIS PROGRAM:

NO : NUMBER OF CASES TO BE PLOTTED BY THE PLOTTING ROUTINE. IF THERE ARE NO PLOTS THIS FIELD IS LEFT BLANK.

STATE : IS A VECTOR OF THE UPPER AND LOWER SERVICEABILITY BOUNDS FOR EACH STATE.

QZERO : ARE THE INITIAL STATE PROBABILITIES WHICH MEASURE THE QUALITY CONTROL ON THE SYSTEM.

N : IS THE NUMBER OF TERMS IN THE DIRICHLET SERIES FOR THE SYSTEM RESPONSE FUNCTION OBTAINED BY LEAST SQUARES FITTING TECHNIQUES.

L : IS THE MAXIMUM NUMBER OF TIME INTERVALS OVER WHICH THE SYSTEM IS TO BE STUDIED.

C MAIN0001
C MAIN0002
C MAIN0003
C MAIN0004
C MAIN0005
C MAIN0006
C MAIN0007
C MAIN0008
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C MAIN0010
C MAIN0011
C MAIN0012
C MAIN0013
C MAIN0014
C MAIN0015
C MAIN0016
C MAIN0017
C MAIN0018
C MAIN0019
C MAIN0020
C MAIN0021
C MAIN0022
C MAIN0023
C MAIN0024
C MAIN0025
C MAIN0026
C MAIN0027
C MAIN0028
C MAIN0029
C MAIN0030
C MAIN0031
C MAIN0032
C MAIN0033
C MAIN0034
C MAIN0035
C MAIN0036

OF SERVICEABILITY, RELIABILITY, AND LIFE EXPECTANCY ARE PROVIDED.
 IN ADDITION, THE PROGRAM DETERMINES THE DENSITY FUNCTION AND THE
 CUMULATIVE DISTRIBUTION OF THE LIFE OF THE SYSTEM. FURTHER, STATE
 AND MARGINAL PROBABILITIES FOR THE SYSTEM ARE DETERMINED BY THE
 PROGRAM TO PROVIDE INPUTS TO THE PREDICTION OF MAINTENANCE LEVEL
 REQUIRED THROUGHOUT THE LIFE OF THE SYSTEM.

IMPLICIT REAL*8 (A-H,C-\$)
 INTEGER*4 TIME(48)
 REAL*8 K1(48),K2(48)
 REAL*8 LAMBDA
 DIMENSION EXPVAL(4,48),VAR(4,48),RESP(48),VARESP(48)
 DIMENSION W(48),PERICC(6),TIMEIN(3),SPANS(3)
 COMMON ERRD,VARRD,VART(60)
 COMMON G(20),DEL(20),VARE,LAMBDA,A,VARA,D,VARD,T(60),TD,TO,DOF
 COMMON V1,V2, RQ,ER,VARR,EK(60),VAREK(60),DT(60)
 COMMON GAM(60),PI,PI2,GRAD(20),B,C,ESV,VARSV,CHANL,VARER,N,L,IRUT
 COMMON /MAINDT/ VARK1(48), VARK2(48),TF(48)
 COMMON /MAINSV/ PS(48,10),P(48),VARP(48),RELY(48),FT(48),PT(48)
 COMMON /MAINSV/ PSUM(48),STATE(20),IPLT,LAST,IPLOUT
 COMMON /MAINPR/ PM(10,10),QZERO(10)
 COMMON /SERVPL/ EL,KZ(6),KAA
 DATA PERIOD/' DAYS ',' MONTHS ',' QUARTERS ',' HALVES ',' YEARS '
 1,'PERIODS '/,TIMEIN/' ',' TIME ',' IN '/
 DATA SPANS/' DAY ',' MONTH ',' YEAR '/
 CALL ERRSET (208,256,-1,1)
 KAA=-100
 LAST=0
 PI=3.14159265
 PI2=9.8696044
 KOUNT=1
 READ (5,4441) NO
 READ (5,4444)(STATE(IT), IT=1,20)
 READ (5,4444) (QZERO(IT), IT=1,10)
 READ (5,100,END=3) N,L,IRUT,IPLT,IPLOUT
 IF (IPLOUT.LE.0) IPLCUT=7

MAIN0073
 MAIN0074
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 MAIN0090
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 MAIN0100
 MAIN0101
 MAIN0102
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 MAIN0107
 MAIN0108

```

READ (5,100) (TIME(IT),IT=1,L)
READ (5,105)(G(I), I=1,N)
READ (5,105)(GRAD(I), I=1,N)
READ (5,105)(DEL(I), I=1,N)
LA=TIME(L)
READ (5,105) (W(I), I=1,12)
READ (5,105) VARE,LAMEDA,A,VARA,D,VARD,TD,TO,V1,V2,RO,B,C,CHANL,
1VARER,DOF,H1,H2,AST,PSI
NOWTD=TD+.5

```

~~IF (NOWTD.NE.TD) GO TO 74~~ — IF (NOWTD.EQ.T.1) ~~GO~~ ~~NOWTD=1~~ GO TO 74

```

DO 73 I=1,LA
K=0

```

```

T(I)=0.
DO 72 J=1,NOWTD
K=K*(1-K/12)+1

```

72 T(I)=T(I)+W(K)

73 T(I)=T(I)/TD

GO TO 75
74 DO 745 I=1,LA
745 T(I)=W(I)

~~74 CONTINUE~~

CHANGE TIME UNITS INTO MONTHS. 75 CONTINUE

```

IF (NO.EQ.1) LAST=1

```

```

NOW=6

```

```

IF (TD.LE..034.AND.TD.GE..032) NOW=1

```

```

IF (TD.EQ.1.) NOW=2

```

```

IF (TD.EQ.3.) NOW=3

```

```

IF (TD.EQ.6.) NOW=4

```

```

IF (TD.EQ.12.) NOW=5

```

```

NEW=2

```

```

IF (TD.LT.1.) NEW=1

```

```

IF (TD.GT.12.) NEW=3

```

```

STD=TD

```

```

IF (NEW.EQ.1) STD=TD*30.44

```

```

IF (NEW.EQ.3) STD=TD/12.

```

```

SUMG=0.

```

```

SUMGR=0.

```

```

DO 2271 I=1,N

```

```

SUMG=SUMG+G(I)

```

MAIN0109
 MAIN0110
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 MAIN0112
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 MAIN0114
 MAIN0115
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 MAIN0123
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 MAIN0125
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```

2271 SUMGR=SUMGR+GRAD(I)
      ICAB=0
      ICBC=0
      SVARE=DSQRT(VARE)
      SVARER=DSQRT(VARER)
      NAW=1
      NBW=2
      IF (NOW.NE.6) GO TO 1123
      NAW=2
      NBW=3
1123 IF (N.GT.25) ICAB=1
      IF (LA.GT.22) ICBC=1
      K=L
      L=TIME(L)
      LA=K
      DO 71 I=1,L
      TF(I)=1.8*(T(I)-273.)+32.
      K1(I)=10**(-6.-.15*(106.-TF(I)))
      K2(I)=2.+(120.-TF(I))*0.040
      VARK1(I)=(V2*K1(I))**2
      VARK2(I)=(V1*K2(I))**2
71   TREF=1.8*(TO-273.)+32.
      WRITE(6,1234) KOUNT,H1,H2,ICAB,(I,G(I),DEL(I),GRAD(I),DEL(I),I=1,N)
      WRITE (6,1235) SUMG,SUMGR,SVARE,SVARER,B,C,TIMEIN(NAW),TIMEIN(NBW)
      IF (NOW.EQ.6) WRITE (6,1236) STD,SPANS(NEW)
      IF (NOW.NE.6) WRITE (6,1571)
      WRITE (6,1237) PERIOD(NOW),(I,TF(I),K1(I),VARK1(I),K2(I),VARK2(I),
1   I=1,L)
      WRITE (6,1238) RO,ICBC,AST,PSI,A,VARA,D,VARD,LAMBDA,TIMEIN(NAW),TI
1MEIN(NBW)
      IF (NOW.EQ.6) WRITE (6,1236) STD,SPANS(NEW)
      IF (NOW.NE.6) WRITE (6,1571)
      DO 395 I=1,N
395  DEL(I)=DEL(I)*2.63D6
      VARD=VARD/6.9169D12
      D=D/2.63D6

```

```

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MAIN0179
MAIN0180

```

```

CALL INITAL
WRITE (6,1240) PERIOD(NOW),(I,TF(I),GAM(I), I=1,L)
WRITE (6,1241) TREF
DO 31 J=1,3
GO TO (8,9,10), J
8 DO 12 M=1,K
L=TIME(M)
CALL GNRES
I1=1
EXPVAL(1,M)=ERRD
VAR(1,M)=VARRD
EXPVAL(3,M)=ESV
12 VAR(3,M)=VARSV
GO TO 25
9 CALL STRAIN
I1=4
DO 371 I=1,K
EXPVAL(4,I)=EK(TIME(I))
209 371 VAR(4,I)=VAREK(TIME(I))
GO TO 31
10 CALL CRACKS (K1,K2)
I1=2
DO 372 I=1,K
EXPVAL(2,I)=DT(TIME(I))
372 VAR(2,I)=VARDT(TIME(I))
25 DO 221 I=1,K
RESP(I)=EXPVAL(I1,I)
221 VARESP(I)=VAR(I1,I)
IF (IPLT.NE.0) CALL PLOTIT (K,TIME,RESP,VARESP,0,IPLT,IPLOUT)
IF (I1.NE.1) GO TO 31
I1=3
GO TO 25
31 CONTINUE
CALL SERVICE (K,TIME,EXPVAL,VAR,VARE,TD)
WRITE (6,4771) KOUNT
WRITE (6,1470)

```

```

MAIN0181
MAIN0182
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MAIN0194
MAIN0195
MAIN0196
MAIN0197
MAIN0198
MAIN0199
MAIN0200
MAIN0201
MAIN0202
MAIN0203
MAIN0204
MAIN0205
MAIN0206
MAIN0207
MAIN0208
MAIN0209
MAIN0210
MAIN0211
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MAIN0213
MAIN0214
MAIN0215
MAIN0216

```

```

WRITE (6,1471) TIMEIN(NAW),TIMEIN(NBW)
IF (NOW.EQ.6) WRITE (6,1472) STD,SPANS(NEW)
IF (NOW.NE.6) WRITE (6,1242)
WRITE (6,1474) PERIOD(NOW),(TIME(I),EXPVAL(1,I),VAR(1,I),EXPVAL(3,
1 I),VAR(3,I),EXPVAL(4,I),VAR(4,I),EXPVAL(2,I),VAR(2,I), I=1,K)
WRITE (6,4147) TIMEIN(NAW),TIMEIN(NBW)
IF (NOW.EQ.6) WRITE (6,1472) STD,SPANS(NEW)
IF (NOW.NE.6) WRITE (6,1242)
WRITE (6,4148) PERIOD(NOW),(TIME(I),P(I),VARP(I),RELY(I),FT(I),PT(
1 I), I=1,K)
WRITE (6,7771)
IF (NOW.EQ.6) WRITE (6,1472) STD,SPANS(NEW)
WRITE (6,7741) PERIOD(NOW),(IAD, IAD=1,9),(TIME(IAE),(PS(IAE,IAF),
1 IAF=1,10),PSUM(IAE), IAE=1,K)
S1=STATE(1)
S20=STATE(20)
STATE(1)=5.03
STATE(20)=0.
WRITE (6,7742) (STATE(I), I=1,19,2),(STATE(IAG), IAG=2,20,2)
STATE(1)=S1
STATE(20)=S20
WRITE (6,4982) EL
IF (K.LT.10) GO TO 3541
WRITE (6,750)
DO 30 I=1,10
30 WRITE (6,300) (PM(I,J),J=1,10)
3541 CONTINUE
KOUNT=KOUNT+1
WRITE (6,777)
NO=NO-1
GO TO 1
3 CALL EXIT
100 FORMAT (20I4)
105 FORMAT(5E16.7)
300 FORMAT ('0',10F13.7)
750 FORMAT ('1',T45,'T R A N S I T I O N',4X,'P R O B A B I L I T Y',

```

```

MAIN0217
MAIN0218
MAIN0219
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MAIN0221
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MAIN0224
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MAIN0230
MAIN0231
MAIN0232
MAIN0233
MAIN0234
MAIN0235
MAIN0236
MAIN0237
MAIN0238
MAIN0239
MAIN0240
MAIN0241
MAIN0242
MAIN0243
MAIN0244
MAIN0245
MAIN0246
MAIN0247
MAIN0248
MAIN0249
MAIN0250
MAIN0251
MAIN0252

```

14X,'M A T R I X'/T45,'- - - - -',4X,'- - - - -'
 2 -',4X,'- - - - -'/////)
 777 FORMAT (////////1H0,132(1H*))
 1234 FORMAT ('1',T55,'INPUT DATA FOR RUN',I3/1X,T55,21('*')////////1X,'A.
 1 SYSTEM GEOMETRY'///7X,'THE DEPTH OF THE SURFACE LAYER IS',F8.4,'
 2 INCHES'///7X,'THE DEPTH OF THE BASE LAYER IS',F8.4,' INCHES'////////
 3I1,'B. MECHANICAL PROPERTIES'////////1X,T31,'NORMAL DEFLECTION SERIE
 4S (INCHES)',T89,'RADIAL STRAIN SERIES (INCHES/INCH)'/1X,T30,'AT TH
 5E SURFACE BELOW THE LCAD CENTER',T84,'AT THE FIRST INTERFACE UNDER
 6 THE LOAD CENTER'/1X,T30,36('-'),T84,44('-')//1X,T8,'I',T23,
 S 'COEFFICIENTS
 7(G(I))',5X,'EXPONENTS (DELTA(I) 1./SEC)',T80,'COEFFICIENTS (GRAD(I
 8))',5X,'EXPONENTS (DELTA(I) 1./SEC)'/1X,T8,'-',T23,19('-'),5X,27(
 9'-'),T80,22('-'),5X,27('-')//(1X,T8,I2,T23,E16.7,12X,E16.7,T81,E16
 T.7,14X,E16.7))
 1235 FORMAT (//1X,T23,'THE SUM OF THE COEFFICIENTS IS',E14.7,T80,'THE S
 1UM OF THE COEFFICIENTS IS',E14.7//1X,T23,'THE SPATIAL COEFFICIENT
 2OF VARIATION IS',E14.7,T80,'THE SPATIAL COEFFICIENT OF VARIATION I
 3S',E14.7//1X,T28,'SPATIAL CORRELATION PARAMETERS: B =', F7.4,'
 4 C =', F7.4/'1',T56,'FATIGUE PROPERTIES'////////1X,T8,A8,19X,'TEMPERA
 5TURE',23X,'CCEFFICIENT K1',28X,'EXPONENT K2'/1X,T8,A8,23X,'IN',28X
 6,14('-'),28X,11('-'))
 1236 FORMAT (1X,T7,F4.1,A8)
 1237 FORMAT ('+',T36,'DEGREES'/1X,T8,A8,19X,'FEHRENHEIT',20X,'MEAN',11X
 1,'VARIANCE',18X,'MEAN',11X,'VARIANCE'/1X,T8,8('-'),19X,11('-'),19X
 2,4('-'),11X,8('-'),18X,4('-'),11X,8('-')//(1X,T8,I5,18X,F13.4,14X,
 3E16.7,E17.7,8X,E16.7,E17.7))
 1238 FORMAT (///1X,T8,'THE CORRELATION COEFFICIENT OF K1 AND K2 IS',F7.
 14 //////////I1,'C. LOAD CHARACTERISTICS'////////1X,T8, 'RADIUS OF THE LO
 2AD =',F8.4,' INCHES'///1X,T8,'MEAN LOAD INTENSITY =',F8.4,' PSI'//
 3//1X,T61,'MEAN',20X,'VARIANCE'/1X,T61,4('-'),20X,8('-')//1X,T8,'L
 4OAD AMPLITUDE (INTENSITY MULTIPLIES)',E25.7,E26.7//1X,T8,'DURATION
 5 OF THE LOADS (SECONDS)',E31.7,E26.7////////1X,T8,'MEAN RATE OF (POISS
 6ON) TRAFFIC LOADS:',F8.0,' LOADS/MONTH'/'1', 'D. TEMPERATURES'///
 S/
 71X,T8,A8,30X,'TEMPERATURE',34X,'TEMPERATURE SHIFT'/1X,T8,A8,34X,'I

MAIN0253
 MAIN0254
 MAIN0255
 MAIN0256
 MAIN0257
 MAIN0258
 MAIN0259
 MAIN0260
 MAIN0261
 MAIN0262
 MAIN0263
 MAIN0264
 MAIN0265
 MAIN0266
 MAIN0267
 MAIN0268
 MAIN0269
 MAIN0270
 MAIN0271
 MAIN0272
 MAIN0273
 MAIN0274
 MAIN0275
 MAIN0276
 MAIN0277
 MAIN0278
 MAIN0279
 MAIN0280
 MAIN0281
 MAIN0282
 MAIN0283
 MAIN0284
 MAIN0285
 MAIN0286
 MAIN0287
 MAIN0288


```

SUBROUTINE INITAL
IMPLICIT REAL*8 (A-H,C-$)
REAL*8 LAMBDA
COMMON ERRO,VARRO,VARCT(60)
COMMON G(20),DEL(20),VARE,LAMBDA,A,VARA,D,VARD,T(60),TD,TO,DOF
COMMON V1,V2,RO,ER,VARR,EK(60),VAREK(60),DT(60)
COMMON GAM(60),PI,PI2,GRAD(20),B,C,ESV,VARSV,CHANL,VARER,N,L,IRUT
COMMON /INITGR/ ALPHA(20,48),BE(20,48),PHI(20,48)
DO 500 K=1,L
TEMP=.162*(T(K)-TO)
TEM=DABS(TEMP)
GAM(K)=10.**TEM
IF (TEMP.LT.0) GAM(K)=1./GAM(K)
500 CONTINUE
SVARD=DSQRT(VARD)
DO 4 K=1,L
DO 4 I=1,N
SIK=GAM(K)*DEL(I)/2.
SIKD=SIK*D
HSIN=DSINH(SIKD)
SSQ=SIKD*SIKD/(2.*PI2)
BSSQ=1./(1.+SSQ)
PHI(I,K)=HSIN*BSSQ+((HSIN*(PI2*(1.+SSQ))-2.+8.*SSQ*BSSQ)-4.*SIKD*
1 DCOSH(SIKD))*0.5*VARD*SIK*SIK/PI2*BSSQ**2
ALPHA(I,K)=HSIN*BSSQ
4 BE(I,K)=SVARD*(SIK*DCCSH(SIKD)*BSSQ-2.*SIKD*SIK*HSIN/(PI2*BSSQ**2)
1)
RETURN
END

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INIT0029

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C SUBROUTINE GNRES
 C THIS SUBROUTINE DETERMINES THE GENERAL RESPONSE OF THE SYSTEM TO
 C A RANDOM SET OF INPUTS IN TERMS OF LOAD CHARACTERISTICS, MATERIALS
 C CHARACTERISTICS, AND TEMPERATURE HISTORIES. THE OUTPUT OF THIS
 C SUBROUTINE IS EXPRESSED IN PROBABILISTIC ESTIMATES OF THE RESPONSE
 C OF THE SYSTEM IN TERMS OF THE FOLLOWING INDICATORS:
 C 1. RUT DEPTH IN INCHES
 C 2. SLOPE VARIANCE AS DEFINED BY THE AASHO ROAD TEST EXPRESSION,
 C 3. STRESSES, STRAINS, OR DEFORMATIONS AT ANY POINT WITHIN THE
 C SYSTEM AT ANY TIME PERIOD.
 C THE OUTPUTS ARE EXPRESSED IN TERMS OF THE MEANS AND VARIANCES OF
 C THE ABOVE PARAMETERS. THE ROUTINE USES THE FOLLOWING INPUTS FROM
 C THE MAIN EXECUTIVE PROGRAM: THE RATE OF LOAD APPLICATIONS IN THE
 C FORM OF POISSON MODE, THE MEANS AND VARIANCES OF THE AMPLITUDES OF
 C THESE LOADS, THE MEANS AND VARIANCES OF THE DURATION OF LOADS, THE
 C RATE OF CHANNELIZED TRAFFIC IN SOME CENTRAL CHANNEL WHICH CAUSES
 C RUTTING, THE DEGREES OF FREEDOM DEFINING THE NUMBER OF CHANNELS IN
 C WHICH THE TRAFFIC CAN POSSIBLY MOVE ACROSS ONE LANE IN A HIGHWAY,
 C AND A TEMPERATURE TEMPLATE DEFINING THE HISTORY OF TEMPERATURES
 C IN A DESIRED RANGE OF TIME. THESE TEMPERATURES ARE USED INDIRECTLY
 C BY THIS PROGRAM IN THE FORM OF TEMPERATURE SHIFT FACTORS $GAM(K)$ 'S.
 C ALSO INPUTS TO THIS ROUTINE, ARE SETS OF RESPONSE FUNCTIONS TO A
 C STATIC LOAD EFFECTS IN THE FORM OF MEANS AND VARIANCES OF THE
 C COEFFICIENTS OF AN EXPONENTIAL SERIES REPRESENTATION OF THIS
 C RESPONSE, AND THE EXPONENTS OF THIS SERIES. FOR THE RUT DEPTH AND
 C SLOPE VARIANCE COMPUTATIONS, THE RESPONSE FUNCTIONS ARE EXPRESSED
 C IN TERMS OF THE VERTICAL DEFLECTIONS AT THE SURFACE UNDER THE
 C CENTER OF THE APPLIED LOADS.
 C THE OUTPUTS OF THIS ROUTINE ARE NORMALLY USED IN THE COMPUTATION
 C OF THE SERVICEABILITY OF THE SYSTEM AT ANY DESIRED TIME PERIOD.
 C
 C IMPLICIT REAL*8(A-H,O-S)
 C INTEGER P,Q
 C REAL*8 IK1,IK2,IK3
 C REAL*8 LAMBDA,LAM
 C DIMENSION V(20,48),VW(20,48),S(20,48)

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 GNRS0034
 GNRS0035
 GNRS0036

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COMMON ERRD,VARRD,VARCT(60)
COMMON G(20),DEL(20),VARE,LAMBDA,A,VARA,D,VARD,T(60),TD,TO,DCF
COMMON V1,V2,RO,ER,VARR,EK(60),VAREK(60),DT(60)
COMMON GAM(60),PI,PI2,GRAD(20),B,C,ESV,VARSV,CHANL,VARER,N,L,IRUT
COMMON /INITGR/ ALPHA(20,48),BE(20,48),PHI(20,48)
LAM=LAMBDA*CHANL-LAMBDA/DOF
IF (IRUT.NE.1) LAM=LAMBDA
SVARD=DSQRT(VARD)
SIGMA=DSQRT(VARA)
BETA=T(L)/TO
ED=0.0
DO 15 K=1,L
ERK=0.
SUM=0.
IF(K.EQ.L)GO TO 5
KA=K+1
DO 2 P=KA,L
SUM=SUM+GAM(P)
CONTINUE
DO 10 I=1,N
S(I,K)=GAM(K)*DEL(I)
SIK=S(I,K)/2.
V(I,K)=DEXP(-TD*DEL(I)*(GAM(K)+SUM))
VW(I,K)=TD
IF (SIK.NE.0.) VW(I,K)=(DEXP(-SUM*TD*DEL(I))-V(I,K))/S(I,K)
IF(K.EQ.L.AND.TD*GAM(K)*DEL(I).GT.80.) VW(I,K)=0.0
BC=VW(I,K)*PHI(I,K)*G(I)
ERK=ERK+BC
CONTINUE
ERK=ERK*LAM*A
15 ED=ED+ERK
ER=-ED
ER=ER*BETA
ERRD=ER
VARA=0.
VARRK=0.0

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GNRS0037
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GNRS0060
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GNRS0064
GNRS0065
GNRS0066
GNRS0067
GNRS0068
GNRS0069
GNRS0070
GNRS0071
GNRS0072

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```

ESV=0.0
VARSV=0.0
VARR=0.0
DO 30 K=1,L
IK1=0.0
IK2=0.0
RSZK=0.0
SUMN=0.
IF (K.EQ.L) GO TO 7177
JAA=K+1
DO 5645 JAB=JAA,L
5645 SUMN=SUMN+GAM(JAB)
7177 CONTINUE
DO 20 I=1,N
TERM1=G(I)*G(I)*V(I,K)*V(I,K)
TERM2=TERM1
IF (S(I,K).NE.0.) TERM2=G(I)*G(I)/(2.*S(I,K))*(DEXP(-2.*SUM*DEL(I)
1*TD)-V(I,K)*V(I,K))
IF (2.*S(I,K)*TD.GT.100.) TERM2=0.0
221 IK1=IK1+TERM2*ALPHA(I,K)**2
IK2=IK2+TERM2*BE(I,K)**2
RSZK=RSZK+G(I)*VW(I,K)*PHI(I,K)
IF(I.EQ.N)GO TO 21
SUM1=0.0
SUM2=0.0
JA=I+1
DO 22 J=JA,N
TERM1=G(I)*G(J)*V(I,K)*V(J,K)
225 TERM2=G(I)*G(J)/(S(I,K)+S(J,K))*(DEXP(-SUM*(DEL(I)+DEL(J))*TD)-V(I
1,K)*V(J,K))
IF((S(I,K)+S(J,K))*TD.GT.100.) TERM2=0.0
226 SUM1=SUM1+TERM2*ALPHA(I,K)*ALPHA(J,K)
SUM2=SUM2+TERM2*BE(I,K)*BE(J,K)
22 CONTINUE
IK1=IK1+SUM1*2.
IK2=IK2+SUM2*2.

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GNRS0073
GNRS0074
GNRS0075
GNRS0076
GNRS0077
GNRS0078
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GNRS0080
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GNRS0090
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GNRS0094
GNRS0095
GNRS0096
GNRS0097
GNRS0098
GNRS0099
GNRS0100
GNRS0101
GNRS0102
GNRS0103
GNRS0104
GNRS0105
GNRS0106
GNRS0107
GNRS0108

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20  CONTINUE
21  CONTINUE
    IK3=RSZK*RSZK
    VARRK=LAM*((VARE+1.)*((VARA+A*A)*(IK1+IK2)+LAM*A*A*IK3)-LAM*A*A*IK
13)
    ZK=RSZK*LAMBDA*A
    VARZK=LAMBDA*(VARA+A*A)*(IK1+IK2)
    EZ2K=VARZK+LAMBDA*LAMEDA*A*A*IK3
    EZ4K=ZK**4+6.*ZK*ZK*VARZK+3.*VARZK*VARZK
    VARZ2K=EZ4K-EZ2K*EZ2K
    VARSV=VARSV+VARZ2K
    COV=0.0
    COV1=0.0
    IF(K.EQ.L)GO TO 27
    MAAAAA=K+1
    DO 25 M=MAAAAA,L
    SUM=0.0
    SUMM=0.
    IF (M.EQ.L) GO TO 1
    JAC=M+1
    DO 3145 JAD=JAC,L
3145 SUMM=SUMM+GAM(JAD)
1  CONTINUE
    DO 24 I=1,N
    IF(I.EQ.N)GO TO 25
    JA=I+1
    DO 24 J=JA,N
    SIK=S(I,K)
    SJM=S(J,M)
    AMU=(D*D*SIK*SIK+D*D*SJM*SJM)/(4.*PI2)
    ANU=(D*D*SIK*SJM/(4.*PI2))**2
    AKIJ=(SIK+SJM)*D/2.
    EPS=(SIK-SJM)*D/2.
    FI=1./(1.+AMU+ANU)
    AJ1=DSINH(AKIJ)
    AJ2=DCOSH(AKIJ)

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GNRS0109
GNRS0110
GNRS0111
GNRS0112
GNRS0113
GNRS0114
GNRS0115
GNRS0116
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GNRS0120
GNRS0121
GNRS0122
GNRS0123
GNRS0124
GNRS0125
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GNRS0136
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GNRS0139
GNRS0140
GNRS0141
GNRS0142
GNRS0143
GNRS0144

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	AJ3=DSINH(EPS)	GNRS0145
	AJ4=DCOSH(EPS)	GNRS0146
	GAMKM=2.*AMU/D+4.*ANU/D	GNRS0147
	DELKM=2.*AMU/(D*D)+12.*ANU/(D*D)	GNRS0148
	XSI=(AKIJ**2*AJ2-EPS**2*AJ4)*FI/2.-(AKIJ*AJ1-EPS*AJ3)*GAMKM*FI*FI	GNRS0149
	1+(AJ2-AJ4)*DELKM*FI*FI*(DELKM*FI-0.5)	GNRS0150
	SUMT1=G(I)*G(J)/(SIK+SJM)*(DEXP(-TD*(SUMN*DEL(I)+SUMM*DEL(J)))-V(I	GNRS0151
	1,K)*V(J,M))	GNRS0152
	SUMTV=(AJ2-AJ4)*FI*0.5	GNRS0153
24	SUM=SUM+SUMT1*(SUMTV+C.5*XSI*VARD)	GNRS0154
	COV1=COV1+SUM*LAM*(VARA+A*A)	GNRS0155
25	COV=COV+SUM*LAMBDA*(VARA+A*A)	GNRS0156
27	CONTINUE	GNRS0157
	ESV=ESV+EZ2K*VARE+2.*CCV	GNRS0158
30	VARR=VARR+VARRK+2.0*CCV1*(1.0+VARE)	GNRS0159
	VARR=VARR*BETA*BETA	GNRS0160
	VARRD=VARR	GNRS0161
	VARSV=VARSV*4.*B*B*BETA**4/(C**4)	GNRS0162
	ESV=ESV*2.*B*BETA*BETA/(C*C)	GNRS0163
	RETURN	GNRS0164
	END	GNRS0165


```

EK(K)=A*SUM/2.+A*VARD*SUM1/4.
DO 12 I=1,N
ALPHA(I)=1.+DEXP(-GAM(K)*DEL(I)*D)
12 BET(I)=1.+(GAM(K)*DEL(I)*D)**2/(4.*PI2)
SUM=0.0
SUM1=0.0
SUM2=0.0
SUM3=0.0
DO 15 I=1,N
SUM=SUM+GRAD(I)*ALPHA(I)/BET(I)
15 SUM2=SUM2+GRAD(I)*(GAM(K)*DEL(I)/2.*(ALPHA(I)-1.)/BET(I)+2.*D*(GAM
1(K)*DEL(I)/(2.*PI))**2*ALPHA(I)/BET(I)**2)
C VARIANCE OF TENSILE STRAIN.
20 VAREK(K)=SUM*SUM*(VARA+VARER*A*A)/4.+SUM2*SUM2*VARD*A*A/4.
RETURN
END

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STRN0037
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STRN0049
STRN0050
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STRN0052

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SUBROUTINE CRACKS (K1,K2)

THIS SUBROUTINE DETERMINES THE AREA OF CRACKING IN A PAVEMENT STRUCTURE IN SQUARE YARDS PER 1000 SQUARE YARDS. IT UTILIZES A FATIGUE LAW TO DETERMINE THE NUMBER OF LOADS TO FAILURE AT A RANDOM STRAIN LEVEL AS DETERMINED BY SUBROUTINE STRAIN. THE NUMBER OF LOADS TO FAILURE IS DETERMINED IN A PROBABILISTIC FASHION AND IS EXPRESSED IN TERMS OF MEANS AND VARIANCES. THE PROGRAM THEN USES A MINER'S CRITERION TO DETERMINE THE LEVEL OF DAMAGE CAUSED BY CRACKING WITHIN THE SYSTEM AT ANY POINT IN TIME. THIS IS ALSO EXPRESSED IN A STOCHASTIC FORM IN TERMS OF MEANS AND VARIANCES. THE OUTPUT OF THIS SUBROUTINE IS USED IN THE DETERMINATIONS OF THE SERVICEABILITY INDEX OF THE SYSTEM AT ANY POINT IN TIME.

IMPLICIT REAL*8 (A-H,C-\$)

REAL*8 K1(48),K2(48),LAMBDA

COMMON ERRD,VARRD,VARCT(60)

COMMON G(20),DEL(20),VARE,LAMBDA,A,VARA,C,VARD,T(60),TD,TO,DOF

COMMON V1,V2, RO,ER,VARR,EK(60),VAREK(60),DT(60)

COMMON GAM(60),PI,PI2,GRAD(20),B,C,ESV,VARSV,CHANL,VARER,N,L,IRUT

COMMON /MAINDT/ VARK1(48), VARK2(48),TF(48)

ADT=0.

AVDT=0.

DO 10 K=1,L

THE FOLLOWING IS A PREADJUSTMENT TO THE FATIGUE EQUATION.

EK(K)=EK(K)/1.0002

VAREK(K)=VAREK(K)/1.0004

SA=DSQRT(VARK2(K))

SC=DSQRT(VARK1(K))

COEK=DABS(1./EK(K))

DLG=DLOG(COEK)

EXPECTED NUMBER OF LOADS TO FAILURE

RNK=COEK**K2(K)*(K1(K)+RO*SC*SA*DLG+.5*K1(K)*VARK2(K)*DLG*DLG+.5*K1(K)*K2(K)*(K2(K)+1.)*COEK**2*VAREK(K))

VARIANCE OF NUMBER OF LOADS TO FAILURE

VARNK=COEK**K2(K) *{(VARK1(K)+K1(K)*K1(K)*DLG*DLG*VARK2(K)+K2(K

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CRAK0033

CRAK0034

CRAK0035

CRAK0036

	1)*K2(K)*K1(K)*K1(K)*VAREK(K)+2.*K1(K)*DLG*RO*SC*SA)*OOEK**K2(K)	CRAK0037
C	EXPECTED VALUE FOR DAMAGE DUE TO CRACKING	CRAK0038
	DT(K)=ADT+LAMBDA*TD/RNK+LAMBDA*TD/RNK*VARNK/RNK/RNK	CRAK0039
	ADT=DT(K)	CRAK0040
C	VARIANCE OF DAMAGE DUE TO CRACKING	CRAK0041
	XX=LAMBDA*TD/RNK/RNK	CRAK0042
	VARDT(K)=AVDT+XX*(1.+XX*VARNK)	CRAK0043
	AVDT=VARDT(K)	CRAK0044
10	CONTINUE	CRAK0045
C	AREA OF CRACKING PER 1000 SQUARE YARDS.	CRAK0046
	DO 20 K=1,L	CRAK0047
	DT(K)=1000.*DT(K)	CRAK0048
20	VARDT(K)=1000000.*VARDT(K)	CRAK0049
	RETURN	CRAK0050
	END	CRAK0051

```

C      SUBROUTINE SERVICE (K,TIME,EXPVAL,VAR,VARE,TD)
C
C      THIS SUBROUTINE COMPUTES THE EXPECTED VALUES AND VARIANCES FOR THE
C      SERVICEABILITY INDEX AT DIFFERENT POINTS IN TIME. IT ALSO COMPUTES
C      THE MARGINAL STATE PROBABILITIES, I.E., THE PROBABILITIES OF THE
C      DIFFERENT LEVELS OF SERVICEABILITY INDEX AT ANY TIME GIVEN ITS
C      MEANS AND VARIANCES AT THAT TIME.
C      THE INPUTS TO THIS PROGRAM ARE THE VALUES OF THE DAMAGE COMPONENTS
C      DETERMINED PROBABILISTICALLY IN SUBROUTINES GNRES AND DAMAGE AT
C      DIFFERENT TIME POINTS. ALSO INPUT TO THIS PROGRAM ARE THE UPPER
C      AND LOWER BOUNDS OF THE DIFFERENT LEVELS OF SERVICEABILITY TO BE
C      DEFINED AS DESIRED. THE PROGRAM USES THE AASHO PSI EQUATION TO
C      DETERMINE THE SERVICEABILITY INDEX FOR THE SYSTEM.
C
      IMPLICIT REAL*8 (A-H,C-$)
      INTEGER*4 ST(10),TIME(48)
      DIMENSION DEV(48),EXPVAL(4,48),VAR(4,48),A(10,11)
      DIMENSION PPS(48)
      COMMON /MAINSV/ PS(48,10),P(48),VARP(48),RELY(48),FT(48),PT(48)
      COMMON /MAINSV/ PSUM(48),STATE(20),IPLT,LAST,IPOUT
      COMMON /SERVPL/ EL,KZ(6),KAA
      KAA=K
      DO 201 NA=1,10
      ST(NA)=NA
201  DEV(NA)=0.
      KN=0
      KA=1+(K-1)/10
      NF=K/KA
      KB=K-NF*KA
      KC=KA-KB
      DO 141 KD=1,KC
141  KZ(KD)=NF
      IF (KB.EQ.0) GO TO 143
      DO 142 KE=1,KB
142  KZ(KD+KE)=NF+1
143  JZ=0

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KW=1
EL=0.
KNOW=0
C WW IS THE BASE 10 LOGARITHM OF E.
WW=1.91*0.4342944
DO 101 I=1,K
KNOW=KNOW+1
DEV(I)=0.
C THE FOLLOWING FOUR STATEMENTS ARE INTENDED TO AMPLIFY THE EFFECTS
C OF RUTTING AND ROUGHNESS IN THE SERVICEABILITY EQUATION.
EXPVAL = EXPVAL(1,I)*1.0D05
VAR1=VAR(1,I)*1.0D10
EXPVA3 = EXPVAL(3,I)*1.0D09
VAR3=VAR(3,I)*1.0D18
E=1.+EXPVA3
P(I)=5.03-1.91*DLOG10(E)-0.01*DSQRT(EXPVAL(2,I))-1.38*EXPVAL**2-
10.5*(WW*VAR3/E**2+0.0125*EXPVAL(2,I)**(-1.5)*VAR(2,I)+1.38*VAR1)
C224 THIS IS TO KEEP THE SERVICEABILITY INDEX PHYSICALLY MEANINGFUL.
IF(P(I).LT.0.0) P(I)=0.0
VARP(I)=0.5*((WW/E)**2*VAR3+0.000025*VAR(2,I)/EXPVAL(2,I)+7.6176*
1EXPVAL**2*VAR1)+P(I)*VARE
C THE LAST PARAMETER IN THE ABOVE EQUATION REPRESENTS AN INITIAL
C QUALITY CONTROL PARAMETER.
STDEVP=DSQRT(VARP(I))
PSUM(I)=0.
DO 100 NA=1,20,2
U1=(STATE(NA)-P(I))/STDEVP
U2=(STATE(NA+1)-P(I))/STDEVP
CALL DNDTR (U2,P2,G)
CALL DNDTR (U1,P1,F)
P12=P1-P2
C THE FOLLOWING RESTRICTIONS ARE INTENDED TO AVOID ERRORS DUE TO
C SOME NUMERICAL OSCILLATIONS IN THE COMPUTATIONS.
IF(P12.GT.1.0) P12=1.0
IF(P12.LT.0.0) P12=0.0
PSUM(I)=PSUM(I)+P12

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      IF (KNOW.LE.10) A((NA+1)/2,KNOW)=P12
      PPS((NA+1)/2)=P12
      PS(I,(NA+1)/2)=P12
100  CONTINUE
      FT(I)=PS(I,10)
      PT(I)=FT(I)
      IF(FT(I).GT.1.0) FT(I)=1.0
      IF (I.NE.1) PT(I)=FT(I)-FT(I-1)
      IF (PT(I).LT.0.) PT(I)=0.
      EL=EL+DFLOAT(TIME(I))*PT(I)
      RELY(I)=1.-PS(I,10)
      JZ=JZ+1
      IF (JZ.LE.KZ(KW)) GO TO 500
      KW=KW+1
      JZ=1
500  CONTINUE
225  101  IF (IPLT.NE.0) CALL PLOTIT (10,ST,PPS,DEV,I,IPLT,IPOUT)
      IF (IPLT.NE.0) CALL PLOTIT (K,TIME,P,VARP,0,IPLT,IPOUT)
      EL=EL*TD/12.
      IF (IPLT.NE.0) CALL PLOTIT (K,TIME,FT,DEV,-3,IPLT,IPOUT)
      IF (IPLT.NE.0) CALL PLOTIT (K,TIME,FT,DEV,-2,IPLT,IPOUT)
      IF (IPLT.NE.0) CALL PLOTIT (K,TIME,PT,DEV,-2,IPLT,IPOUT)
      JLAST=1000*LAST-2
      IF (IPLT.NE.0) CALL PLOTIT (K,TIME,RELY,DEV,JLAST,IPLT,IPOUT)
      IF (KNOW.GE.10) CALL PROB (A)
      RETURN
      END

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SERV0090
SERV0091
SERV0092
SERV0093
SERV0094
SERV0095
SERV0096
SERV0097
SERV0098
SERV0099

```

SUBROUTINE DNDTR (X,P,D)

THIS SUBROUTINE DETERMINES THE CUMULATIVE DISTRIBUTION AND THE
PROBABILITY MASS FUNCTION FOR A NORMALLY DISTRIBUTED RANDOM
VARIABLE. IN THIS CASE THE RANDOM VARIABLE IS THE SERVICEABILITY
OF THE SYSTEM AT DIFFERENT POINTS IN TIME. THE INPUTS TO THIS
PROGRAM ARE MEANS AND VARIANCES OF THE SERVICEABILITY AT DIFFERENT
TIME POINTS. THE OUTPUTS ARE THE PROBABILITY MASS FUNCTION AND
THE DISTRIBUTION AT EACH TIME PERIOD.
THIS IS SIMPLY A SUBSTIUTE FOR THE NORMAL DISTRIBUTION TABLES.

IMPLICIT REAL*8 (A-H,C-\$)

AX=DABS(X)

T=1.0/(1.0+.2316419*AX)

D=0.3989423*DEXP(-X*X/2.0)

P=1.0-D*T*(((1.330274*T-1.821256)*T+1.781478)*T-0.3565638)*T+0.31
193815)

IF (X.LT.0.) P=1.0-P

RETURN

END

NDTR0001
NDTR0002
NDTR0003
NDTR0004
NDTR0005
NDTR0006
NDTR0007
NDTR0008
NDTR0009
NDTR0010
NDTR0011
NDTR0012
NDTR0013
NDTR0014
NDTR0015
NDTR0016
NDTR0017
NDTR0018
NDTR0019
NDTR0020

C
C
C
C
C
C
C
C
C
C

SUBROUTINE PROB (A)

THIS SUBROUTINE DETERMINES THE TRANSITION PROBABILITIES FOR A
HOMOGENEOUS MARKOV PRCESS GIVEN ITS MARGINAL STATE PROBABILITIES
AT DIFFERENT TIME POINTS. FOR AN N-STATE PROCESS, THIS PROGRAM
UTILIZES AN N SET OF STATE PROBABILITIES AT N CONSECUTIVE TIME
PERIODS PLUS THE INITIAL STATE PROBABILITIES. THE OUTPUT OF THIS
PROGRAM IS AN N BY N UPPER TRIANGULAR MATRIX. IT IS ASSUMED THAT
THE MARKOV PROCESS HERE IS ONE OF CONTINUOUS DETERIORATION.

IMPLICIT REAL*8 (A-H,C-\$)
DIMENSION Q(10,11),A(10,11)
COMMON /MAINPR/ PM(10,10),QZERO(10)
CALL ERRSET (209,256,-1,1)

DO 1111 I=1,10

1111 Q(I,1)=QZERO(I)

EPS=0.1D-15

DO 5 I=2,11

IN1=I-1

DO 5 J=1,10

Q(J,I)=A(IN1,J)

DO 66 I=2,10

IN1=I-1

DO 66 J=1,IN1

PM(I,J)=100000.

PM(1,1)=Q(1,2)/Q(1,1)

DO 20 N=2,10

IN1=N+1

DO 10 J=2,IN1

JN1=J-1

A(JN1,IN1)=Q(N,J)

DO 10 K=1,N

A(JN1,K)=Q(K,JN1)

DETER=1.

DO 9 K=1,N

DETER=DETER*A(K,K)

PROB0001
PROB0002
PROB0003
PROB0004
PROB0005
PROB0006
PROB0007
PROB0008
PROB0009
PROB0010
PROB0011
PROB0012
PROB0013
PROB0014
PROB0015
PROB0016
PROB0017
PROB0018
PROB0019
PROB0020
PROB0021
PROB0022
PROB0023
PROB0024
PROB0025
PROB0026
PROB0027
PROB0028
PROB0029
PROB0030
PROB0031
PROB0032
PROB0033
PROB0034
PROB0035
PROB0036

```

IF (DABS(A(K,K)).LT.EPS) WRITE (6,200) A(K,K),EPS,DETER
KP1=K+1
DO 6 J=KP1,IN1
6  A(K,J)=A(K,J)/A(K,K)
  A(K,K)=1.
  DO 9 I=1,N
    IF (I.EQ.K.OR.A(I,K).EQ.0.) GO TO 9
    DO 8 J=KP1,IN1
8    A(I,J)=A(I,J)-A(I,K)*A(K,J)
    A(I,K)=0.
9    CONTINUE
    DO 20 J=1,N
20   PM(J,N)=A(J,IN1)
      RETURN
200  FORMAT (///'OPIVOT =',G14.7,', THE MINIMUM ALLOWABLE VALUE IS',
1G8.1,', THE DETERMINANT IS NOW',G14.7,', THE MATRIX MAY BE SIN
2GULAR'//)
      END

```

```

PROB0037
PROB0038
PROB0039
PROB0040
PROB0041
PROB0042
PROB0043
PROB0044
PROB0045
PROB0046
PROB0047
PROB0048
PROB0049
PROB0050
PROB0051
PROB0052
PROB0053
PROB0054

```


SUBROUTINE PLOTIT (NO,TIME,LINE,DEV,ICNTRL,IPLT,IPOUT)

THIS SUBROUTINE WRITES THE DATA FOR THE PLOTTING ROUTINE ONTO
EITHER CARDS OR SCRATCH DISK.

INTEGER*4 TIME(48)

REAL*8 LINE(48),DEV(48),EL

DATA ICN1/0/,ICN2/0/

COMMON /SERVPL/ EL,KZ(6),KAA

IF (IPOUT.NE.7) WRITE (IPOUT) NO,ICNTRL,IPLT,KAA

IF (IPOUT.EQ.7) WRITE (7,100) NO,ICNTRL,IPLT,KAA

IF (IPOUT.NE.7) WRITE (IPOUT) (TIME(I),LINE(I),DEV(I), I=1,NO)

IF (IPOUT.EQ.7) WRITE (7,200) (TIME(I),LINE(I),DEV(I), I=1,NO)

IF (ICNTRL.EQ.-2) ICN2=ICN2+1

IF (ICN2.NE.2) GO TO 1

ICN2=0

IF (IPOUT.NE.7) WRITE (IPOUT) EL

IF (IPOUT.EQ.7) WRITE (7,300) EL

IF (ICNTRL.GT.0.AND.ICNTRL.LE.900) ICN1=ICN1+1

IF (ICN1.EQ.1.AND.IPLCUT.NE.7) WRITE (IPOUT) KZ

IF (ICN1.EQ.1.AND.IPLCUT.EQ.7) WRITE (7,100) KZ

IF (ICN1.EQ.KAA) ICN1=0

FORMAT (6I4)

FORMAT (2(I4,2E16.7))

FORMAT (E16.7)

RETURN

END

PLOT0001

PLOT0002

PLOT0003

PLOT0004

PLOT0005

PLOT0006

PLOT0007

PLOT0008

PLOT0009

PLOT0010

PLOT0011

PLOT0012

PLOT0013

PLOT0014

PLOT0015

PLOT0016

PLOT0017

PLOT0018

PLOT0019

PLOT0020

PLOT0021

PLOT0022

PLOT0023

PLOT0024

PLOT0025

PLOT0026

PLOT0027

C
C
C
C

229
1

100
200
300

```

      INTEGER*4 TIME(48)
      REAL*8 LINE(48),DEV(48),EL
      DATA ICN1/0/,ICN2/0/
10    READ (5,100,END=1000) IPLTIN
      IF (IPLTIN.LE.0) IPLTIN=5
      IF (IPLTIN.NE.5) READ (IPLTIN) NO,ICNTRL,IPLT,KAA
      IF (IPLTIN.EQ.5) READ (5,100) NO,ICNTRL,IPLT,KAA
      IF (IPLTIN.NE.5) READ (IPLTIN) (TIME(I),LINE(I),DEV(I), I=1,NO)
      IF (IPLTIN.EQ.5) READ (5,200) (TIME(I),LINE(I),DEV(I), I=1,NO)
      IF (ICNTRL.EQ.-2) ICN2=ICN2+1
      IF (ICN2.NE.2) GO TO 1
      ICN2=0
      IF (IPLTIN.NE.5) READ (IPLTIN) EL
      IF (IPLTIN.EQ.5) READ (5,300) EL
1    IF (ICNTRL.GT.0.AND.ICNTRL.LE.900) ICN1=ICN1+1
      IF (ICN1.EQ.1.AND.IPLTIN.NE.5) READ (IPLTIN) KZ
      IF (ICN1.EQ.1.AND.IPLTIN.EQ.5) READ (5,100) KZ
      IF (ICN1.EQ.KAA) ICN1=0
230  CALL PLOTS (NO,TIME,LINE,DEV,ICNTRL,IPLT,EL,KAA)
      GO TO 10
1000 CALL EXIT
100  FORMAT (6I4)
200  FORMAT (2(I4,2E16.7))
300  FORMAT (E16.7)
      END

```

```

GRAF0001
GRAF0002
GRAF0003
GRAF0004
GRAF0005
GRAF0006
GRAF0007
GRAF0008
GRAF0009
GRAF0010
GRAF0011
GRAF0012
GRAF0013
GRAF0014
GRAF0015
GRAF0016
GRAF0017
GRAF0018
GRAF0019
GRAF0020
GRAF0021
GRAF0022
GRAF0023
GRAF0024
GRAF0025

```

SUBROUTINE PLOTS (NO,TIME,LINE,DEV,ICNTRL,IPLT,EL,KAA)

THIS ROUTINE PLOTS GRAPHS ON A STROMBERG CARLSON 4020 PLOTTER.
THESE GRAPHS ARE GENERATED FROM DATA PASSED BY THE CALLING ROUTINE IN THE
FORM OF THE NUMBER OF POINTS, THE TIMES AT WHICH THE SYSTEM IS ANALYZED
AT, THE EXPECTED VALUE AT EACH POINT AND THE VARIANCE. THIS DATA IS THEN
ACTED UPON IN SEVERAL WAYS, DEPENDING ON THE VALUE OF THE CONTROL
VARIABLE. IN A NORMAL CALLING SCHEME A GRAPH WILL BE GENERATED WITH
LABELS AND BACKGROUND GRID. PLOTTED ON THE GRAPH WILL BE THE EXPECTED
VALUE AND TWO OTHER LINES WHICH REPRESENT THAT VALUE PLUS OR MINUS THE
STANDARD DEVIATION AT EACH POINT. IN OTHER SCHEMES SINGLE LINES CAN BE
DRAWN OR MULTIPLE PLOTS CAN APPEAR ON A SINGLE GRAPH.

IMPLICIT REAL*8 (A-H,C-Z), INTEGER*2 (\$)

INTEGER*4 TIME(48)

REAL*8 DEV(48),LINE(48),LINES(3,48)

DIMENSION IDIST(2),IDNO(3),IFACTR(3),LENGTH(10,4),\$EADIN(2000)

COMMON /SERVPL/ EL,KZ(12)

DATA IFRAME/2/,IQ/0/,KLATCH/0/,NEW/0/,ICN2/0/

ILAST=0

IF (ICNTRL.LE.900) GO TO 110

ICNTRL=ICNTRL-1000

ILAST=1

110 IF (IQ.GT.0) GO TO 111

READ (5,750)(IDNO(JC), JC=1,3), NUMB

CALL STOIDV (IDNO,NUMB,3)

111 DO 220 IDIOT=1,NO

UPDOWN=DSQRT(DEV(IDIOT))

LINES(2,IDIOT)=LINE(IDIOT)

LINES(1,IDIOT)=LINES(2,IDIOT)+UPDOWN

220 LINES(3,IDIOT)=LINES(2,IDIOT)-UPDOWN

ANY=1.

IF (ICNTRL.LE.0) NEW=C

IF (ICNTRL.EQ.-2) ICN2=ICN2+1

IQ=IQ+1

LAST=NO

PLTS0001
PLTS0002
PLTS0003
PLTS0004
PLTS0005
PLTS0006
PLTS0007
PLTS0008
PLTS0009
PLTS0010
PLTS0011
PLTS0012
PLTS0013
PLTS0014
PLTS0015
PLTS0016
PLTS0017
PLTS0018
PLTS0019
PLTS0020
PLTS0021
PLTS0022
PLTS0023
PLTS0024
PLTS0025
PLTS0026
PLTS0027
PLTS0028
PLTS0029
PLTS0030
PLTS0031
PLTS0032
PLTS0033
PLTS0034
PLTS0035
PLTS0036

C
C
C
C
C
C
C
C
C
C
C

231

IF (ICNTRL.EQ.-3) GO TO 813
 AKEEPH=LINES(1,1)
 AKEEPL=LINES(3,1)
 DO 221 IOTA=1,NO
 AKEEPL=DMIN1(AKEEPL,LINES(3,IOTA))
 221 AKEEPH=DMAX1(AKEEPH,LINES(1,IOTA))
 IF (NEW.GT.0) GO TO 813
 IFLAG=0
 READ (5,753) LATCH,AX,AY,LOGX,LOGY,NOTE
 IF (AX.EQ.0.) AX=10.
 IF (AY.LE.0) AY=10.
 IF (LATCH.LT.0) GO TO 812
 IF (KLATCH.GT.0) GO TO 809
 KLATCH=LATCH
 IZ=0
 MOMENT=1
 KNT=1
 KMT=40
 809 IZ=IZ+1
 232 READ (5,751)(LENGTH(IZ,IE), IE=1,4)
 DO 811 IE=1,4
 IF (LENGTH(IZ,IE).LE.C) GO TO 811
 ICH=(LENGTH(IZ,IE)-1)/80+1
 KONSTN=LENGTH(IZ,IE)-(LENGTH(IZ,IE)-1)/80*80
 DO 810 IB=1,ICH
 READ (5,752)(\$EADIN(IA), IA=KNT,KMT)
 MCT=40
 IF (IB.EQ.ICH) MCT=(KONSTN-1)/2+1
 KNT=KNT+MCT
 810 KMT=KMT+MCT
 811 CONTINUE
 GO TO 813
 812 MOMENT=1
 IF (KLATCH.EQ.0) GO TO 814
 KLATCH=0
 IY=IZ

PLTS0037
 PLTS0038
 PLTS0039
 PLTS0040
 PLTS0041
 PLTS0042
 PLTS0043
 PLTS0044
 PLTS0045
 PLTS0046
 PLTS0047
 PLTS0048
 PLTS0049
 PLTS0050
 PLTS0051
 PLTS0052
 PLTS0053
 PLTS0054
 PLTS0055
 PLTS0056
 PLTS0057
 PLTS0058
 PLTS0059
 PLTS0060
 PLTS0061
 PLTS0062
 PLTS0063
 PLTS0064
 PLTS0065
 PLTS0066
 PLTS0067
 PLTS0068
 PLTS0069
 PLTS0070
 PLTS0071
 PLTS0072

```

      IZ=0
814  IZ=IZ*(1-IZ/IY)+1
813  CALL SMXYV (LOGX,LOGY)
      IF (ICNTRL.EQ.-4) GO TO 889
      MW=0
      NW=0
      I=1
      J=1
      NX=3
      NY=-6
      IF (ICNTRL.LE.0.AND.ICNTRL.NE.-2.AND.ICNTRL.NE.-3) GO TO 223
      NY=3
      AKEEPH=1.
      AKEEPL=0.
      IF (ICNTRL.EQ.-2) GO TO 223
      IF (ICNTRL.EQ.-3) GO TO 274
      IFLAG=1
      NEW=NEW+1
      IF (NEW.GT.1) GO TO 223
      NEWS=0
      NEWZ=1
233  IF (NEW.GT.1) GO TO 171
      NOWL=TIME(1)
      NOWH=TIME(1)
      DO 222 IB=1,NO
      NOWL=MINO(NOWL,TIME(IE))
222  NOWH=MAXO(NOWH,TIME(IE))
      IF (NOWL.GE.NOWH) GO TO 887
      IF (AKEEPL.EQ.AKEEPH) GO TO 887
      CX=(DFLOAT(NOWH)-DFLOAT(NOWL))/AX
      DY=(AKEEPH-AKEEPL)/AY
      DO 450 IB=1,3
      IF (LENGTH(IZ,IB).LE.C) GO TO 449
      IFACTR(IB)=754/(6*LENGTH(IZ,IB))
      IF (IFACTR(IB)-3) 447,450,448
447  IFACTR(IB)=3

```

```

PLTS0073
PLTS0074
PLTS0075
PLTS0076
PLTS0077
PLTS0078
PLTS0079
PLTS0080
PLTS0081
PLTS0082
PLTS0083
PLTS0084
PLTS0085
PLTS0086
PLTS0087
PLTS0088
PLTS0089
PLTS0090
PLTS0091
PLTS0092
PLTS0093
PLTS0094
PLTS0095
PLTS0096
PLTS0097
PLTS0098
PLTS0099
PLTS0100
PLTS0101
PLTS0102
PLTS0103
PLTS0104
PLTS0105
PLTS0106
PLTS0107
PLTS0108

```

448 IF (IFACTR(IB).LE.6) GO TO 450
 IFACTR(IB)=6
 GO TO 450
 449 IFACTR(IB)=0
 450 CONTINUE
 NARGIN=1014-12*IFACTR(3)
 IR=(LENGTH(IZ,3)-1)/46+1
 CALL SETMIV (12*IFACTR(2),0,12*IFACTR(1),12*IFACTR(3)+18*(IR-1)+20
 (*IFLAG)
 IF (IQ.EQ.1) IFRAME=2
 NEWK=0
 451 CALL GRIDIV (IFRAME,DFLOAT(TIME(1)),DFLOAT(TIME(NO)),AKEEPL,AKEEPH
 1,DX,DY,NW,MW,-I,J,NX,NY)
 IFRAME=4
 IF (ICNTRL.GT.0) MARGIN=524-90*KZ(NEWZ)/2
 DO 791 IC=1,3
 IF (IFACTR(IC).EQ.0) GO TO 791
 IF (IC.LT.3) GO TO 144
 IF (IR.EQ.1) GO TO 144
 IDIST(1)=12*IFACTR(2)+126
 GO TO 147
 144 IDIST(1)=(1024+12*IFACTR(2)-6*IFACTR(IC)*LENGTH(IZ,IC))/2
 IF (IC.EQ.2) IDIST(1)=(1064+12*IFACTR(1)-12*IFACTR(3)-26*IR-6*IFAC
 1TR(2)*LENGTH(IZ,2))/2
 147 KONSTN=512*(IC-1)*(IC-2)
 LLAMA=-IC*IC+3*IC-1
 IDIST(2)=KONSTN+6*LLAMA*IFACTR(IC)
 KT=-90*(IC-1)*(IC-3)
 NS=-IC*IC+4*IC-2
 NT=3-NS
 CALL CHSIZV (IFACTR(IC),IFACTR(IC))
 CALL RITSTV (6*IFACTR(IC),26)
 DO 455 IB=1,3
 455 CALL RITE2V (IDIST(NS),IDIST(NT),1023,KT,1,LENGTH(IZ,IC),MOMENT,\$E
 1ADIN,IERR)
 791 MOMENT=MOMENT+2*LENGTH(IZ,IC)-LENGTH(IZ,IC)/2*2

PLTS0109
 PLTS0110
 PLTS0111
 PLTS0112
 PLTS0113
 PLTS0114
 PLTS0115
 PLTS0116
 PLTS0117
 PLTS0118
 PLTS0119
 PLTS0120
 PLTS0121
 PLTS0122
 PLTS0123
 PLTS0124
 PLTS0125
 PLTS0126
 PLTS0127
 PLTS0128
 PLTS0129
 PLTS0130
 PLTS0131
 PLTS0132
 PLTS0133
 PLTS0134
 PLTS0135
 PLTS0136
 PLTS0137
 PLTS0138
 PLTS0139
 PLTS0140
 PLTS0141
 PLTS0142
 PLTS0143
 PLTS0144

```

IF (LENGTH(IZ,4).LE.0) GO TO 171
IRS=(LENGTH(IZ,4)-1)/40+1
MCT=12*IFACTR(2)+126
DO 117 JE=1,IRS
117 CALL PRINTV (40,$EADIN,MCT,972-12*IFACTR(3)-18*(IR-1)-16*JE,MOMENT
1+40*(JE-1))
MOMENT=MOMENT+2*LENGTH(IZ,4)-LENGTH(IZ,4)/2*2
171 IF (ICNTRL.LE.0) GO TO 274
NEWS=NEWS+1
IF (NEWS.LE.KZ(NEWZ)) GO TO 273
NEWK=NEWK+NEWS-1
NEWS=0
NEWZ=NEWZ+1
MOMENT=1
GO TO 451
273 NEWF=NEWK+NEWS
NEWT=NEWF
IF (NEWF.EQ.27) NEWT=63
IF (NEWF.EQ.48) NEWT=62
IF (NEWF.EQ.59) NEWT=0
IF (NEWF.EQ.13) NEWT=61
LLAMA=NEWF/10
KONSTN=NEWF-10*LLAMA
KKK=0
IF (LLAMA.GT.0) KKK=1
CALL POINTV (MARGIN,NARGIN,-48,ANY)
CALL POINTV (MARGIN+12,NARGIN,NEWT,ANY)
CALL POINTV (MARGIN+24,NARGIN,-48,ANY)
CALL POINTV (MARGIN+36,NARGIN,-11,ANY)
CALL POINTV (MARGIN+48,NARGIN,-48,ANY)
IF (LLAMA.GT.0) CALL POINTV (MARGIN+60,NARGIN,-LLAMA,ANY)
CALL POINTV (MARGIN+12*KKK+60,NARGIN,-KONSTN,ANY)
CALL POINTV (MARGIN+12*KKK+72,NARGIN,-48,ANY)
MARGIN=MARGIN+12*KKK+84
274 KONSTN=1
DO 272 JD=1,NO

```

```

PLTS0145
PLTS0146
PLTS0147
PLTS0148
PLTS0149
PLTS0150
PLTS0151
PLTS0152
PLTS0153
PLTS0154
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PLTS0156
PLTS0157
PLTS0158
PLTS0159
PLTS0160
PLTS0161
PLTS0162
PLTS0163
PLTS0164
PLTS0165
PLTS0166
PLTS0167
PLTS0168
PLTS0169
PLTS0170
PLTS0171
PLTS0172
PLTS0173
PLTS0174
PLTS0175
PLTS0176
PLTS0177
PLTS0178
PLTS0179
PLTS0180

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236

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272 IF (NYV(LINES(1,JD))-NYV(LINES(3,JD)).GT.20) KONSTN=4
    IF (NOTE.EQ.1) GO TO 119
    IF (ICNTRL.LE.0) NEWT=38
    IF (ICNTRL.EQ.-3) NEWT=43
    DO 270 IE=1,NO
    IF (KONSTN.EQ.1) GO TO 270
    CALL POINTV (DFLOAT(TIME(IE)),LINES(1,IE),63)
    CALL POINTV (DFLOAT(TIME(IE)),LINES(2,IE),55)
270 CALL POINTV (DFLOAT(TIME(IE)),LINES(3,IE),NEWT)
119 IF (NOTE.EQ.2) GO TO 889
    DO 271 JB=2,NO
    JA=JB-1
    NOW1=NXV(DFLOAT(TIME(JA)))
    NOW2=NXV(DFLOAT(TIME(JB)))
    DO 271 JC=1,3
    MY1=NYV(LINES(JC,JA))
    MY2=NYV(LINES(JC,JB))
    THETA=DATAN(DFLOAT(MY2-MY1)/DFLOAT(NOW2-NOW1))
    IAX=7.*DCOS(THETA)+.5
    IAY=7.*DSIN(THETA)+.5
    IAX1=.7*DFLOAT(IAX)
    IAY1=.7*DFLOAT(IAY)
    DO 271 JD=1,KONSTN
271 CALL LINEV (NOW1+IAX1,MY1+IAY1,NOW2-IAX,MY2-IAY)
    GO TO 889
887 NNT=(IFACTR(1)+1)/2
    WRITE (6,771) ($EADIN(NNU), NNU=1,NNT)
    NNT=(IFACTR(2)+1)/2
    WRITE (6,772) ($EADIN(NNU), NNU=1,NNT)
    WRITE (6,773)
    IF (IQ.EQ.1) IFRAME=2
889 IF (ICN2.NE.2) GO TO 891
    DO 890 I=1,4
890 CALL LINEV (NXV(EL),NYV(AKEEPL),NXV(EL),NYV(AKEEPH))
891 IF (ILAST.EQ.1) CALL PLTND (NUMB)
    RETURN

```

```

PLTS0181
PLTS0182
PLTS0183
PLTS0184
PLTS0185
PLTS0186
PLTS0187
PLTS0188
PLTS0189
PLTS0190
PLTS0191
PLTS0192
PLTS0193
PLTS0194
PLTS0195
PLTS0196
PLTS0197
PLTS0198
PLTS0199
PLTS0200
PLTS0201
PLTS0202
PLTS0203
PLTS0204
PLTS0205
PLTS0206
PLTS0207
PLTS0208
PLTS0209
PLTS0210
PLTS0211
PLTS0212
PLTS0213
PLTS0214
PLTS0215
PLTS0216

```


750 FORMAT (3A4,I4)
751 FORMAT (4I4)
752 FORMAT (40A2)
753 FORMAT (I4,2F4.0,3I4)
771 FORMAT (/////'OUNABLE TO PLOT ',40A2)
772 FORMAT (17X,'VERSUS'/16X,40A2)
773 FORMAT ('OMAXIMUM VALLE = MINIMUM VALUE')
END

PLTS0217
PLTS0218
PLTS0219
PLTS0220
PLTS0221
PLTS0222
PLTS0223
PLTS0224

APPENDIX III. TABLES OF COMPUTER INPUTS AND OUTPUTS
FOR THE SENSITIVITY ANALYSIS.

INPUT DATA FOR RUN 1

A. SYSTEM GEOMETRY

THE DEPTH OF THE SURFACE LAYER IS 3.0000 INCHES
THE DEPTH OF THE BASE LAYER IS 3.0000 INCHES

B. MECHANICAL PROPERTIES

NORMAL DEFLECTION SERIES (INCHES)
AT THE SURFACE BELOW THE LOAD CENTER

RADIAL STRAIN SERIES (INCHES/INCH)
AT THE FIRST INTERFACE UNDER THE LOAD CENTER

239

I	COEFFICIENTS (G(I))	EXPONENTS (DELTA(I) 1./SEC)	COEFFICIENTS (GRAD(I))	EXPONENTS (DELTA(I) 1./SEC)
1	0.1575696D-02	0.5000000D 03	0.3677329D-04	0.5000000D 03
2	-0.1052930D-01	0.5000000D 02	-0.2761362D-03	0.5000000D 02
3	-0.1125625D 00	0.5000000D 01	-0.3465797D-02	0.5000000D 01
4	-0.7370919D-01	0.5000000D 00	-0.3399192D-02	0.5000000D 00
5	-0.8947927D-01	0.5000000D-01	-0.2937002D-02	0.5000000D-01
6	-0.8419085D-01	0.5000000D-02	-0.2733562D-02	0.5000000D-02
7	-0.8589208D-01	0.5000000D-03	-0.1925807D-02	0.5000000D-03
8	-0.8760840D-01	0.5000000D-04	-0.2475571D-02	0.5000000D-04
9	-0.8049774D-01	0.5000000D-05	-0.2024479D-02	0.5000000D-05
10	-0.9525585D-01	0.5000000D-06	-0.2426863D-02	0.5000000D-06
11	-0.6553936D-01	0.5000000D-07	-0.1786828D-02	0.5000000D-07
12	-0.1331329D 00	0.5000000D-08	-0.3532887D-02	0.5000000D-08
13	0.3953552D-01	0.5000000D-09	0.1020432D-02	0.5000000D-09
14	-0.3897400D 00	0.5000000D-10	-0.1028252D-01	0.5000000D-10
15	0.1577142D 01	0.0	0.4182856D-01	0.0

THE SUM OF THE COEFFICIENTS IS 0.3101158D 00

THE SUM OF THE COEFFICIENTS IS 0.5619121D-02

THE SPATIAL COEFFICIENT OF VARIATION IS 0.2500000D 00

THE SPATIAL COEFFICIENT OF VARIATION IS 0.2500000D 00

SPATIAL CORRELATION PARAMETERS: B = 1.0000 C = 0.5000

TABLE (4.1): ANALYSIS OF SYSTEM 1; THIN GEOMETRY, MEDIUM PROPERTIES, MEDIUM QUALITY CONTROL LEVEL

FATIGUE PROPERTIES

TIME IN YEARS	TEMPERATURE IN DEGREES FAHRENHEIT	COEFFICIENT K1		EXPONENT K2	
		MEAN	VARIANCE	MEAN	VARIANCE
1	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
2	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
3	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
4	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
5	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
6	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
7	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
8	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
9	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
10	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
11	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
12	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
13	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
14	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
15	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00

THE CORRELATION COEFFICIENT OF K1 AND K2 IS 0.0

C. LOAD CHARACTERISTICS

RADIUS OF THE LOAD = 6.4000 INCHES

MEAN LOAD INTENSITY = 80.0000 PSI

	MEAN	VARIANCE
LOAD AMPLITUDE (INTENSITY MULTIPLIES)	0.10000000 01	0.62500000-01
DURATION OF THE LOADS (SECONDS)	0.50000000-01	0.40000000-03

MEAN RATE OF (POISSON) TRAFFIC LOADS: 22500. LOADS/MONTH

TABLE 4.1 -- CONTINUED

D. TEMPERATURES

TIME IN YEARS	TEMPERATURE IN DEGREES FEHRENHEIT	TEMPERATURE SHIFT FACTORS, GAMMAS (FOR THE TIME - TEMPERATURE SUPERPOSITION)
1	74.3000	0.57147880 00
2	74.3000	0.57147880 00
3	74.3000	0.57147880 00
4	74.3000	0.57147880 00
5	74.3000	0.57147880 00
6	74.3000	0.57147880 00
7	74.3000	0.57147880 00
8	74.3000	0.57147880 00
9	74.3000	0.57147880 00
10	74.3000	0.57147880 00
11	74.3000	0.57147880 00
12	74.3000	0.57147880 00
13	74.3000	0.57147880 00
14	74.3000	0.57147880 00
15	74.3000	0.57147880 00

REFERENCE TEMPERATURE = 77.0000

TABLE (4.1) -- CONTINUED

***** OUTPUT DATA FOR RUN 1 *****

RESPONSE HISTORY

TIME IN YEARS	RUT DEPTH		SLOPE VARIANCE		STRAIN		CRACKS	
	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE
1	0.7749276D-05	0.3763186D-11	C.4804291D-09	0.9242749D-20	0.5804028D-02	0.2222222D-05	0.3366507D 02	0.6420968D 02
3	0.1000001D-04	0.3950931D-11	C.5019319D-09	0.9657749D-20	0.5804028D-02	0.2222222D-05	0.1009952D 03	0.1526290D 03
6	0.1149962D-04	0.4029092D-11	C.5081370D-09	0.9777295D-20	0.5804028D-02	0.2222222D-05	0.2019904D 03	0.3852581D 03
10	0.1276699D-04	0.4094688D-11	C.5114631D-09	0.9840995D-20	0.5804028D-02	0.2222222D-05	0.3366507D 03	0.6420968D 03
12	0.1324438D-04	0.4121933D-11	C.5124144D-09	0.9859065D-20	0.5804028D-02	0.2222222D-05	0.4039809D 03	0.7705161D 03
15	0.1381838D-04	0.4159002D-11	C.5133544D-09	0.9876735D-20	0.5804028D-02	0.2222222D-05	0.5049761D 03	0.9631451D 03

PERFORMANCE CHARACTERISTICS

TIME IN YEARS	SERVICEABILITY		RELIABILITY	CUMULATIVE DISTRIBUTION OF LIFE	PROBABILITY MASS FUNCTION OF LIFE
	MEAN	VARIANCE			
1	0.3788061D 01	0.3243014D 00	0.9906087D 00	0.9391311D-02	0.9391311D-02
3	0.3181876D 01	0.3508474D 00	0.8917278D 00	0.1082722D 00	0.9888085D-01
6	0.2691704D 01	0.3726717D 00	0.6535826D 00	0.3460174D 00	0.2377452D 00
10	0.2223838D 01	0.3947016D 00	0.3594884D 00	0.6405116D 00	0.2944942D 00
12	0.2034306D 01	0.4040436D 00	0.2566153D 00	0.7433847D 00	0.1028730D 00
15	0.1795505D 01	0.4162029D 00	0.1552081D 00	0.8447919D 00	0.1014072D 00

TABLE (4.1) -- CONTINUED

M A R G I N A L P R O B A B I L I T I E S

TIME IN YEARS	STATE 1	STATE 2	STATE 3	STATE 4	STATE 5	STATE 6	STATE 7	STATE 8	STATE 9	STATE 10	SUM
1	0.0311068	0.0593343	0.1181527	0.1793798	0.2076465	0.1832770	0.1233429	0.0632872	0.0247547	0.0093913	0.9996733
3	0.0024295	0.0080199	0.0252168	0.0616649	0.1172914	0.1735469	0.1997611	0.1788779	0.1246085	0.1082722	0.9596890
6	0.0002036	0.0009635	0.0041760	0.0142937	0.0385842	0.0821865	0.1381523	0.1832773	0.1918964	0.3460174	0.9997528
10	0.0000146	0.0000921	0.0005230	0.0023720	0.0085989	0.0249168	0.0577157	0.1068764	0.1582286	0.6405116	0.9598496
12	0.0000047	0.0000331	0.0002076	0.0010446	0.0042231	0.0137164	0.0357929	0.0750461	0.1264347	0.7433847	0.9998878
15	0.0000011	0.0000087	0.0000612	0.0003485	0.0016031	0.0059621	0.0179259	0.0435760	0.0856498	0.8447919	0.9999283
UPPER BOUND	5.030000	4.849900	4.549900	4.249900	3.949900	3.649900	3.349900	3.049900	2.749900	2.449900	
LOWER BOUND	4.850000	4.550000	4.250000	3.950000	3.650000	3.350000	3.050000	2.750000	2.450000	0.0	

THE EXPECTED VALUE OF LIFE IS 0.74330320 C1 YEARS

TABLE (4.1) -- CONTINUED

INPUT DATA FOR RUN 2

A. SYSTEM GEOMETRY

THE DEPTH OF THE SURFACE LAYER IS 4.0000 INCHES

THE DEPTH OF THE BASE LAYER IS 6.0000 INCHES

B. MECHANICAL PROPERTIES

NORMAL DEFLECTION SERIES (INCHES)
AT THE SURFACE BELOW THE LOAD CENTER

RADIAL STRAIN SERIES (INCHES/INCH)
AT THE FIRST INTERFACE UNDER THE LOAD CENTER

I	NORMAL DEFLECTION SERIES (INCHES)		RADIAL STRAIN SERIES (INCHES/INCH)	
	COEFFICIENTS (G(I))	EXPONENTS (DELTA(I) 1./SEC)	COEFFICIENTS (GRAD(I))	EXPONENTS (DELTA(I) 1./SEC)
1	0.15515530-03	0.50000000 03	-0.52281250-05	0.50000000 03
2	-0.48290270-02	0.50000000 02	-0.10854580-03	0.50000000 02
3	-0.12828950 00	0.50000000 01	-0.44846940-02	0.50000000 01
4	-0.10641910 00	0.50000000 00	-0.47421340-02	0.50000000 00
5	-0.89415730-01	0.50000000-01	-0.28007080-02	0.50000000-01
6	-0.80044980-01	0.50000000-02	-0.25850370-02	0.50000000-02
7	-0.87355550-01	0.50000000-03	-0.18643220-02	0.50000000-03
8	-0.87062840-01	0.50000000-04	-0.24059860-02	0.50000000-04
9	-0.80667500-01	0.50000000-05	-0.21093450-02	0.50000000-05
10	-0.95264430-01	0.50000000-06	-0.25938150-02	0.50000000-06
11	-0.55577510-01	0.50000000-07	-0.15790460-02	0.50000000-07
12	-0.13276670 00	0.50000000-08	-0.37248130-02	0.50000000-08
13	0.38040160-01	0.50000000-09	0.11301040-02	0.50000000-09
14	-0.38568120 00	0.50000000-10	-0.10673520-01	0.50000000-10
15	0.15599990 01	0.0	0.43371400-01	0.0

THE SUM OF THE COEFFICIENTS IS 0.25482020 00

THE SUM OF THE COEFFICIENTS IS 0.48243100-02

THE SPATIAL COEFFICIENT OF VARIATION IS 0.25000000 00

THE SPATIAL COEFFICIENT OF VARIATION IS 0.25000000 00

SPATIAL CORRELATION PARAMETERS: B = 1.0000 C = 0.5000

TABLE (4.2): ANALYSIS OF SYSTEM 1; WITH MEDIUM GEOMETRY

FATIGUE PROPERTIES

TIME IN YEARS	TEMPERATURE IN DEGREES FAHRENHEIT	COEFFICIENT K1		EXPONENT K2	
		MEAN	VARIANCE	MEAN	VARIANCE
1	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
2	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
3	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
4	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
5	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
6	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
7	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
8	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
9	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
10	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
11	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
12	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
13	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
14	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
15	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00

THE CORRELATION COEFFICIENT OF K1 AND K2 IS 0.0

C. LCAD CHARACTERISTICS

RADIUS OF THE LCAD = 6.4000 INCHES

MEAN LOAD INTENSITY = 80.0000 PSI

	MEAN	VARIANCE
LOAD AMPLITUDE (INTENSITY MULTIPLIES)	0.10000000 01	0.62500000-01
DURATION OF THE LOADS (SECONDS)	0.50000000-01	0.40000000-03

MEAN RATE OF (PCISSON) TRAFFIC LOADS: 22500. LCADS/MONTH

TABLE (4.2) -- CONTINUED

D. TEMPERATURES

TIME IN YEARS	TEMPERATURE IN DEGREES FEHRENHEIT	TEMPERATURE SHIFT FACTORS, GAMMAS (FOR THE TIME - TEMPERATURE SUPERPOSITION)
-----	-----	-----
1	74.3000	0.57147880 00
2	74.3000	0.57147880 00
3	74.3000	0.57147880 00
4	74.3000	0.57147880 00
5	74.3000	0.57147880 00
6	74.3000	0.57147880 00
7	74.3000	0.57147880 00
8	74.3000	0.57147880 00
9	74.3000	0.57147880 00
10	74.3000	0.57147880 00
11	74.3000	0.57147880 00
12	74.3000	0.57147880 00
13	74.3000	0.57147880 00
14	74.3000	0.57147880 00
15	74.3000	0.57147880 00

REFERENCE TEMPERATURE = 77.0000

TABLE(4.2) -- CONTINUED

***** OUTPUT DATA FOR RUN 2 *****

RESPONSE HISTORY

TIME IN YEARS	RUT DEPTH		SLOPE VARIANCE		STRAIN		CRACKS	
	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE
1	0.7749778D-05	0.3763689D-11	C.4804913D-09	0.9258902D-20	0.5036498D-02	0.1701664D-05	0.1899656D 02	0.1597408D 02
3	0.9999285D-05	0.3951299D-11	C.5019728D-09	0.9674155D-20	0.5036498D-02	0.1701664D-05	0.5698968D 02	0.5592225D 02
6	0.1149698D-04	0.4029386D-11	C.5081625D-09	0.9793594D-20	0.5036498D-02	0.1701664D-05	0.1139794D 03	0.1198445D 03
10	0.1276282D-04	0.4094985D-11	C.5114808D-09	0.9857243D-20	0.5036498D-02	0.1701664D-05	0.1899656D 03	0.1597408D 03
12	0.1323982D-04	0.4122250D-11	C.5124307D-09	0.9875313D-20	0.5036498D-02	0.1701664D-05	0.2279587D 03	0.2396890D 03
15	0.1381358D-04	0.4159361D-11	C.5133700D-09	0.9892998D-20	0.5036498D-02	0.1701664D-05	0.2849484D 03	0.2596112D 03

PERFORMANCE CHARACTERISTICS

TIME IN YEARS	SERVICEABILITY		RELIABILITY	CUMULATIVE DISTRIBUTION OF LIFE	PROBABILITY MASS FUNCTION OF LIFE
	MEAN	VARIANCE			
1	0.3802896D 01	0.3252429D 00	0.9911641D 00	0.8835858D-02	0.8835858D-02
3	0.3207369D 01	0.3524246D 00	0.8990123D 00	0.1009877D 00	0.9215179D-01
6	0.2728108D 01	0.3748604D 00	0.6752280D 00	0.3247720D 00	0.2237843D 00
10	0.2271117D 01	0.3975008D 00	0.3883707D 00	0.6116293D 00	0.2868573D 00
12	0.2086126D 01	0.4071057D 00	0.2842927D 00	0.7157073D 00	0.1040775D 00
15	0.1853373D 01	0.4196277D 00	0.1785594D 00	0.8214406D 00	0.1057334D 00

TABLE (4.2) -- CONTINUED

M A R G I N A L P R O B A B I L I T I E S

TIME IN YEARS	STATE 1	STATE 2	STATE 3	STATE 4	STATE 5	STATE 6	STATE 7	STATE 8	STATE 9	STATE 10	SUM
1	0.0331746	0.0619073	0.1214013	0.1816486	0.2073940	0.1806866	0.1201186	0.0609287	0.0235779	0.0088359	0.9996736
3	0.0028289	0.0090294	0.0276540	0.0659430	0.1224458	0.1770605	0.1993986	0.1748848	0.1194547	0.1009877	0.9996874
6	0.0002645	0.0011971	0.0050023	0.0165147	0.0430776	0.0887878	0.1446156	0.1861489	0.1893663	0.3247720	0.9997467
10	0.0000215	0.0001289	0.0006560	0.0030248	0.0104938	0.0291452	0.0648082	0.1153869	0.1645029	0.6116293	0.9998395
12	0.0000074	0.0000489	0.0002514	0.0013955	0.0053781	0.0166788	0.0416245	0.0836018	0.1351438	0.7157073	0.9998775
15	0.0000019	0.0000139	0.0000522	0.0004969	0.0021678	0.0076595	0.0219170	0.0507918	0.0953374	0.8214406	0.9999189
UPPER BOUND	5.030000	4.849900	4.549900	4.249900	3.949900	3.649900	3.349900	3.049900	2.749900	2.449900	
LOWER BOUND	4.850000	4.550000	4.250000	3.950000	3.650000	3.350000	3.050000	2.750000	2.450000	0.0	

THE EXPECTED VALUE OF LIFE IS 0.73315070 01 YEARS

TABLE (4.2) -- CONTINUED

INPUT DATA FOR RUN 3

A. SYSTEM GEOMETRY

THE DEPTH OF THE SURFACE LAYER IS 5.0000 INCHES

THE DEPTH OF THE BASE LAYER IS 9.0000 INCHES

B. MECHANICAL PROPERTIES

NORMAL DEFLECTION SERIES (INCHES)
AT THE SURFACE BELOW THE LOAD CENTER

RADIAL STRAIN SERIES (INCHES/INCH)
AT THE FIRST INTERFACE UNDER THE LOAD CENTER

I	COEFFICIENTS (G(I))	EXPONENTS (DELTA(I) 1./SEC)	COEFFICIENTS (GRAD(I))	EXPONENTS (DELTA(I) 1./SEC)
1	-0.6692105D-03	0.5000000E 03	-0.4773423D-04	0.5000000E 03
2	-0.1390549D-02	0.5000000E 02	0.5654783D-04	0.5000000E 02
3	-0.1356496D 00	0.5000000E 01	-0.5030364D-02	0.5000000E 01
4	-0.1378126D 00	0.5000000E 00	-0.5005978D-02	0.5000000E 00
5	-0.1102247E 00	0.5000000E-01	-0.3340963D-02	0.5000000E-01
6	-0.6697661D-01	0.5000000E-02	-0.2419541D-02	0.5000000E-02
7	-0.6717390E-01	0.5000000E-03	-0.1251593D-02	0.5000000E-03
8	-0.8393347D-01	0.5000000E-04	-0.1684062D-02	0.5000000E-04
9	-0.6711960E-01	0.5000000E-05	-0.1040731D-02	0.5000000E-05
10	-0.8008385D-01	0.5000000E-06	-0.1366615D-02	0.5000000E-06
11	-0.5244350E-01	0.5000000E-07	-0.6689429D-03	0.5000000E-07
12	-0.1163940D 00	0.5000000E-08	-0.2261221D-02	0.5000000E-08
13	0.4872131E-01	0.5000000E-09	0.1884460D-02	0.5000000E-09
14	-0.3670349D 00	0.5000000E-10	-0.8756220D-02	0.5000000E-10
15	0.1457141E 01	0.0	0.3514284D-01	0.0

THE SUM OF THE COEFFICIENTS IS 0.2189558D 00

THE SUM OF THE COEFFICIENTS IS 0.4209883D-02

THE SPATIAL COEFFICIENT OF VARIATION IS 0.2500000D 00

THE SPATIAL COEFFICIENT OF VARIATION IS 0.2500000D 00

SPATIAL CORRELATION PARAMETERS: B = 1.0000 C = 0.5000

TABLE (4.3); ANALYSIS OF SYSTEM 1; WITH THICK GEOMETRY

FATIGUE PROPERTIES

TIME IN YEARS	TEMPERATURE IN DEGREES FAHRENHEIT	COEFFICIENT K1		EXPONENT K2	
		MEAN	VARIANCE	MEAN	VARIANCE
1	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
2	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
3	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
4	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
5	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
6	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
7	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
8	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
9	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
10	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
11	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
12	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
13	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
14	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
15	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00

THE CORRELATION COEFFICIENT OF K1 AND K2 IS 0.0

C. LOAD CHARACTERISTICS

RADIUS OF THE LOAD = 6.4000 INCHES

MEAN LOAD INTENSITY = 80.0000 PSI

	MEAN	VARIANCE
LOAD AMPLITUDE (INTENSITY MULTIPLIES)	0.10000000 01	0.62500000-01
DURATION OF THE LOADS (SECONDS)	0.50000000-01	0.40000000-03

MEAN RATE OF (POISSON) TRAFFIC LOADS: 22500. LOADS/MONTH

TABLE (4.3) -- CONTINUED

D. TEMPERATURES

TIME IN YEARS	TEMPERATURE IN DEGREES FEHRENHEIT	TEMPERATURE SHIFT FACTORS, GAMMAS (FOR THE TIME - TEMPERATURE SUPERPOSITION)
1	74.3000	0.5714788D 00
2	74.3000	0.5714788D 00
3	74.3000	0.5714788D 00
4	74.3000	0.5714788D 00
5	74.3000	0.5714788D 00
6	74.3000	0.5714788D 00
7	74.3000	0.5714788D 00
8	74.3000	0.5714788D 00
9	74.3000	0.5714788D 00
10	74.3000	0.5714788D 00
11	74.3000	0.5714788D 00
12	74.3000	0.5714788D 00
13	74.3000	0.5714788D 00
14	74.3000	0.5714788D 00
15	74.3000	0.5714788D 00

REFERENCE TEMPERATURE = 77.0000

TABLE (4.3) -- CONTINUED

***** OUTPUT DATA FOR RUN 3 *****

RESPONSE HISTORY

TIME IN YEARS	RUT DEPTH		SLOPE VARIANCE		STRAIN		CRACKS	
	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE
1	0.6445996D-05	0.2603907D-11	C.3324200D-09	0.4470871D-20	0.4430322D-02	0.1330973D-05	0.1132923D 02	0.6558356D 01
3	0.8315334D-05	0.2733231D-11	C.3472060D-09	0.4670348D-20	0.4430322D-02	0.1330973D-05	0.3398770D 02	0.2087507D 02
6	0.9594487D-05	0.2789191D-11	C.3517130D-09	0.4731057D-20	0.4430322D-02	0.1330973D-05	0.6767540D 02	0.4175013D 02
10	0.1068111D-04	0.2835968D-11	C.3541559D-09	0.4763782D-20	0.4430322D-02	0.1330973D-05	0.1132923D 03	0.6558356D 02
12	0.1108866D-04	0.2855227D-11	C.3548481D-09	0.4772982D-20	0.4430322D-02	0.1330973D-05	0.1359508D 03	0.8350027D 02
15	0.1157618D-04	0.2881297D-11	C.3555251D-09	0.4781887D-20	0.4430322D-02	0.1330973D-05	0.1699385D 03	0.1043753D 03

PERFORMANCE CHARACTERISTICS

TIME IN YEARS	SERVICEABILITY		RELIABILITY	CUMULATIVE DISTRIBUTION OF LIFE	PROBABILITY MASS FUNCTION OF LIFE
	MEAN	VARIANCE			
1	0.4164723D 01	0.3023784D 00	0.9990910D 00	0.9089796D-03	0.9089756D-03
3	0.3749700D 01	0.3072312D 00	0.9904866D 00	0.9513396D-02	0.8604417D-02
6	0.3406431D 01	0.3115940D 00	0.9566972D 00	0.4330277D-01	0.3378937D-01
10	0.3076679D 01	0.3164256D 00	0.8674117D 00	0.1325883D 00	0.8928546D-01
12	0.2943557D 01	0.3185917D 00	0.8091038D 00	0.1908962D 00	0.5830757D-01
15	0.2776751D 01	0.3215141D 00	0.7178390D 00	0.2821610D 00	0.9126476D-01

TABLE (4.3) -- CONTINUED

M A R G I N A L P R O B A B I L I T I E S
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TIME IN IN YEARS	STATE 1	STATE 2	STATE 3	STATE 4	STATE 5	STATE 6	STATE 7	STATE 8	STATE 9	STATE 10	SUM
1	0.1063439	0.1353850	0.1965557	0.2134602	0.1733980	0.1053556	0.0478755	0.0162680	0.0041327	0.0009090	0.9996874
3	0.0235676	0.0508138	0.1089507	0.1754969	0.2123935	0.1931350	0.1319536	0.0677314	0.0261155	0.0095134	0.9996718
6	0.0048537	0.0153519	0.0451101	0.0996936	0.1661664	0.2089031	0.1981023	0.1417016	0.0764492	0.0433028	0.9996748
10	0.0008095	0.0035974	0.0140848	0.0417640	0.0937627	0.1594280	0.2053283	0.2003090	0.1480199	0.1325883	0.9996938
12	0.0003657	0.0018472	0.0081025	0.0269636	0.0680603	0.1303438	0.1894188	0.2088908	0.1748182	0.1908962	0.9997081
15	0.0001279	0.0007541	0.0038027	0.0145785	0.0424980	0.0542183	0.1588829	0.2038142	0.1988949	0.2821610	0.9997325
UPPER BOUND	5.030000	4.849900	4.549900	4.249900	3.949900	3.649900	3.349900	3.049900	2.749900	2.449900	
LOWER BOUND	4.850000	4.550000	4.250000	3.950000	3.650000	3.350000	3.050000	2.750000	2.450000	0.0	

THE EXPECTED VALUE OF LIFE IS 0.31909800 01 YEARS

TABLE (4.3) -- CONTINUED

INPUT DATA FOR RUN 4

A. SYSTEM GEOMETRY

THE DEPTH OF THE SURFACE LAYER IS 3.0000 INCHES
THE DEPTH OF THE BASE LAYER IS 3.0000 INCHES

B. MECHANICAL PROPERTIES

NORMAL DEFLECTION SERIES (INCHES)
AT THE SURFACE BELOW THE LOAD CENTER

RADIAL STRAIN SERIES (INCHES/INCH)
AT THE FIRST INTERFACE UNDER THE LOAD CENTER

I	COEFFICIENTS (G(I))	EXPONENTS (DELTA(I) 1./SEC)	COEFFICIENTS (GRAD(I))	EXPONENTS (DELTA(I) 1./SEC)
1	-0.63034150-03	0.50000000 03	-0.23399200-04	0.50000000 03
2	-0.34007800-02	0.50000000 02	-0.84068120-04	0.50000000 02
3	-0.17922340 00	0.50000000 01	-0.63007620-02	0.50000000 01
4	-0.17574830 00	0.50000000 00	-0.72541570-02	0.50000000 00
5	-0.14141920 00	0.50000000-01	-0.25702470-02	0.50000000-01
6	-0.14697840 00	0.50000000-02	-0.44579620-02	0.50000000-02
7	-0.11014160 00	0.50000000-03	-0.50808040-02	0.50000000-03
8	-0.12181470 00	0.50000000-04	-0.55326710-02	0.50000000-04
9	-0.10820580 00	0.50000000-05	-0.45763250-02	0.50000000-05
10	-0.13063240 00	0.50000000-06	-0.55284500-02	0.50000000-06
11	-0.88562010-01	0.50000000-07	-0.40297510-02	0.50000000-07
12	-0.18162540 00	0.50000000-08	-0.71830750-02	0.50000000-08
13	0.54031370-01	0.50000000-09	0.68855290-03	0.50000000-09
14	-0.53300480 00	0.50000000-10	-0.18943790-01	0.50000000-10
15	0.21399980 01	0.0	0.75999980-01	0.0

THE SUM OF THE COEFFICIENTS IS 0.27264220 00

THE SUM OF THE COEFFICIENTS IS 0.51230720-02

THE SPATIAL COEFFICIENT OF VARIATION IS 0.25000000 00

THE SPATIAL COEFFICIENT OF VARIATION IS 0.25000000 00

SPATIAL CORRELATION PARAMETERS: B = 1.0000 C = 0.5000

TABLE (4.4): ANALYSIS OF SYSTEM 2; WITH THIN GEOMETRY AND WEAK PROPERTIES

FATIGUE PROPERTIES

TIME IN YEARS	TEMPERATURE IN DEGREES FAHRENHEIT	COEFFICIENT K1		EXPONENT K2	
		MEAN	VARIANCE	MEAN	VARIANCE
1	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
2	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
3	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
4	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
5	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
6	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
7	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
8	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
9	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
10	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
11	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
12	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
13	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
14	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
15	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00

THE CORRELATION COEFFICIENT OF K1 AND K2 IS 0.0

C. LOAD CHARACTERISTICS

RADIUS OF THE LOAD = 6.4000 INCHES

MEAN LOAD INTENSITY = 80.0000 PSI

	MEAN	VARIANCE
LOAD AMPLITUDE (INTENSITY MULTIPLIES)	0.10000000 01	0.62500000-01
DURATION OF THE LOADS (SECONDS)	0.50000000-01	0.40000000-03

MEAN RATE OF (POISSON) TRAFFIC LOADS: 22500. LOADS/MONTH

TABLE (4.4) -- CONTINUED

D. TEMPERATURES

TIME IN YEARS	TEMPERATURE IN DEGREES FEHRENHEIT	TEMPERATURE SHIFT FACTORS, GAMMAS (FOR THE TIME - TEMPERATURE SUPERPOSITION)
1	74.3000	0.5714788D 00
2	74.3000	0.5714788D 00
3	74.3000	0.5714788D 00
4	74.3000	0.5714788D 00
5	74.3000	0.5714788D 00
6	74.3000	0.5714788D 00
7	74.3000	0.5714788D 00
8	74.3000	0.5714788D 00
9	74.3000	0.5714788D 00
10	74.3000	0.5714788D 00
11	74.3000	0.5714788D 00
12	74.3000	0.5714788D 00
13	74.3000	0.5714788D 00
14	74.3000	0.5714788D 00
15	74.3000	0.5714788D 00

REFERENCE TEMPERATURE = 77.0000

TABLE (4.4) -- CONTINUED

***** OUTPUT DATA FOR RUN 4 *****

RESPONSE HISTORY

TIME IN YEARS	ROUT DEPTH		SLOPE VARIANCE		STRAIN		CRACKS	
	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE
1	0.1058124D-04	0.7016140D-11	C.8957364D-09	0.3189094C-19	0.5416499D-02	0.2008280D-05	0.2545644D 02	0.3626053D 02
3	0.1363651D-04	0.7362124D-11	C.9353431D-09	0.3330551D-19	0.5416499D-02	0.2008280D-05	0.7636933D 02	0.1087816D 03
6	0.1567557D-04	0.7507012D-11	C.9468578D-09	0.3371604D-19	0.5416499D-02	0.2008280D-05	0.1527387D 03	0.2175632D 03
10	0.1740845D-04	0.7628664D-11	C.9530464D-09	0.3393539D-19	0.5416499D-02	0.2008280D-05	0.2545644D 03	0.3626053D 03
12	0.1805977D-04	0.7679161D-11	C.9548168D-09	0.3359763D-19	0.5416499D-02	0.2008280D-05	0.3054773D 03	0.4351263D 03
15	0.1884290D-04	0.7747835D-11	C.9565659D-09	0.3405849C-19	0.5416499D-02	0.2008280D-05	0.3818466D 03	0.5439079D 03

PERFORMANCE CHARACTERISTICS

TIME IN YEARS	SERVICEABILITY		RELIABILITY	CUMULATIVE DISTRIBUTION OF LIFE	PROBABILITY MASS FUNCTION OF LIFE
	MEAN	VARIANCE			
1	0.2850049D 01	C.4803975D 00	0.7181406D 00	0.2818594D 00	0.2818594D 00
3	0.1773228D 01	0.6353357D 00	0.1979576D 00	0.8020424D 00	0.5201830D 00
6	0.9048613D 00	C.7625809D 00	0.3842359D-01	0.9615764D 00	0.1595340D 00
10	0.7615952D-01	0.8883939D 00	0.5893929D-02	0.9941061D 00	0.3252966D-01
12	0.0	C.9570301D 00	0.6134792D-02	0.9938652D 00	0.0
15	0.0	0.1050844D 01	0.8426531D-02	0.9915735D 00	0.0

TABLE (4.4) -- CONTINUED

M A R G I N A L P R O B A B I L I T I E S
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TIME IN IN YEARS	STATE 1	STATE 2	STATE 3	STATE 4	STATE 5	STATE 6	STATE 7	STATE 8	STATE 9	STATE 10	SUM
1	0.0019541	0.0051354	0.0146078	0.0345495	0.0679447	0.1111081	0.1510867	0.1708446	0.1606474	0.2818594	0.9997376
3	0.0000567	0.0001906	0.0006566	0.0022133	0.0061133	0.0146783	0.0306377	0.0555927	0.0876939	0.8020424	0.9999156
6	0.0000031	0.0000118	0.0000450	0.0001802	0.0005903	0.0017201	0.0044594	0.0102855	0.0211060	0.9615764	0.9999819
10	0.0000002	0.0000008	0.0000027	0.0000150	0.0000550	0.0001821	0.0005451	0.0014757	0.0036131	0.9941061	0.9999970
12	0.0000004	0.0000013	0.0000053	0.0000200	0.0000684	0.0002127	0.0006031	0.0015573	0.0036632	0.9938652	0.9999970
15	0.0000011	0.0000034	0.0000124	0.0000414	0.0001267	0.0003566	0.0009217	0.0021881	0.0047711	0.9915735	0.9999559
UPPER BOUND	5.030000	4.849900	4.549900	4.249900	3.949900	3.649900	3.349900	3.049900	2.749900	2.449900	
LOWER BOUND	4.850000	4.550000	4.250000	3.950000	3.650000	3.350000	3.050000	2.750000	2.450000	0.0	

THE EXPECTED VALUE OF LIFE IS 0.31249090 01 YEARS

TABLE (4.4) -- CONTINUED

INPUT DATA FOR RUN 5

A. SYSTEM GEOMETRY

THE DEPTH OF THE SURFACE LAYER IS 3.0000 INCHES
THE DEPTH OF THE BASE LAYER IS 3.0000 INCHES

B. MECHANICAL PROPERTIES

I	NORMAL DEFLECTION SERIES (INCHES) AT THE SURFACE BELOW THE LOAD CENTER		RADIAL STRAIN SERIES (INCHES/INCH) AT THE FIRST INTERFACE UNDER THE LOAD CENTER	
	COEFFICIENTS (G(I))	EXPONENTS (DELTA(I) 1./SEC)	COEFFICIENTS (GRAD(I))	EXPONENTS (DELTA(I) 1./SEC)
1	-0.1189495E-01	0.50000000 03	-0.6958951D-04	0.5000000E 03
2	-0.4835758D-02	0.50000000 02	-0.5437924D-04	0.50000000 02
3	-0.8493137D-02	0.50000000 01	-0.7981817D-04	0.5000000E 01
4	-0.1658629D-01	0.50000000 00	-0.2668672D-03	0.50000000 00
5	-0.4478450D-01	0.5000000E-01	-0.1169397D-02	0.5000000E-01
6	-0.2787646D-01	0.50000000-02	-0.8180831D-03	0.5000000E-02
7	-0.4055959D-01	0.5000000E-03	-0.1306233D-02	0.5000000E-03
8	-0.5004829D-01	0.50000000-04	-0.1181159D-02	0.5000000E-04
9	-0.3552848D-01	0.50000000-05	-0.1217805D-02	0.5000000E-05
10	-0.4037762D-01	0.50000000-06	-0.1290623D-02	0.5000000E-06
11	-0.4236698D-01	0.50000000-07	-0.9195805D-03	0.5000000E-07
12	-0.6124020D-01	0.5000000E-08	-0.1436949D-02	0.5000000E-08
13	0.9666992D-02	0.50000000-09	0.9328127D-04	0.5000000E-09
14	-0.1678467D 00	0.50000000-10	-0.3662825D-02	0.5000000E-10
15	0.6688522D 00	0.0	0.1461358D-01	0.0

THE SUM OF THE COEFFICIENTS IS 0.1250802D 00

THE SUM OF THE COEFFICIENTS IS 0.1233553D-02

THE SPATIAL COEFFICIENT OF VARIATION IS 0.2500000D 00

THE SPATIAL COEFFICIENT OF VARIATION IS 0.2500000D 00

SPATIAL CORRELATION PARAMETERS: B = 1.0000 C = 0.5000

TABLE (4.5): ANALYSIS OF SYSTEM 3; WITH THIN GEOMETRY AND STRONG PROPERTIES

FATIGUE PROPERTIES

TIME IN YEARS	TEMPERATURE IN DEGREES FEHRENHEIT	COEFFICIENT K1		EXPONENT K2	
		MEAN	VARIANCE	MEAN	VARIANCE
1	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
2	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
3	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
4	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
5	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
6	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
7	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
8	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
9	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
10	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
11	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
12	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
13	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
14	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00
15	74.3000	0.17579160-10	0.12361070-22	0.38280010 01	0.58614360 00

THE CORRELATION COEFFICIENT OF K1 AND K2 IS 0.0

C. LCAD CHARACTERISTICS

RADIUS OF THE LOAD = 6.4000 INCHES

MEAN LOAD INTENSITY = 80.0000 PSI

	MEAN	VARIANCE
LOAD AMPLITUDE (INTENSITY MULTIPLIES)	0.10000000 01	0.62500000-01
DURATION OF THE LOADS (SECONDS)	0.50000000-01	0.40000000-03

MEAN RATE OF (POISSON) TRAFFIC LOADS: 22500. LCADS/MONTH

TABLE (4.5) -- CONTINUED

D. TEMPERATURES

TIME IN YEARS	TEMPERATURE IN DEGREES FEHRENHEIT	TEMPERATURE SHIFT FACTORS, GAMMAS (FOR THE TIME - TEMPERATURE SUPERPOSITION)
1	74.3000	0.57147880 00
2	74.3000	0.57147880 00
3	74.3000	0.57147880 00
4	74.3000	0.57147880 00
5	74.3000	0.57147880 00
6	74.3000	0.57147880 00
7	74.3000	0.57147880 00
8	74.3000	0.57147880 00
9	74.3000	0.57147880 00
10	74.3000	0.57147880 00
11	74.3000	0.57147880 00
12	74.3000	0.57147880 00
13	74.3000	0.57147880 00
14	74.3000	0.57147880 00
15	74.3000	0.57147880 00

REFERENCE TEMPERATURE = 77.0000

TABLE (4.5) -- CONTINUED

***** OUTPUT DATA FOR RUN 5 *****

R E S P O N S E H I S T O R Y

TIME IN YEARS	RUT DEPTH		SLOPE VARIANCE		STRAIN		CRACKS	
	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE
1	0.3771903D-05	0.8911765D-12	0.1138217D-09	0.4329483D-21	0.1314604D-02	0.1102917D-06	0.8618910D-01	0.3350421D-03
3	0.5035451D-05	0.9487699D-12	0.1206923D-09	0.4552430D-21	0.1314604D-02	0.1102917D-06	0.2585673D 00	0.1005126D-02
6	0.5777328D-05	0.9666030D-12	0.1222212D-09	0.4650916D-21	0.1314604D-02	0.1102917D-06	0.5171346D 00	0.2010252D-02
10	0.6377952D-05	0.9802845D-12	0.1229664D-09	0.4679271D-21	0.1314604D-02	0.1102917D-06	0.8618910D 00	0.3350421D-02
12	0.6604802D-05	0.9858858D-12	0.1231802D-09	0.4687346D-21	0.1314604D-02	0.1102917D-06	0.1034269D 01	0.4020505D-02
15	0.6879354D-05	0.9934504D-12	0.1233934D-09	0.4695332D-21	0.1314604D-02	0.1102917D-06	0.1292837D C1	0.5025631D-02

P E R F O R M A N C E C H A R A C T E R I S T I C S

TIME IN YEARS	SERVICEABILITY		RELIABILITY	CUMULATIVE DISTRIBUTION OF LIFE	PROBABILITY MASS FUNCTION OF LIFE
	MEAN	VARIANCE			
1	0.4734933D 01	0.3008826D 00	0.9959845D 00	0.1552526D-04	0.1552526D-04
3	0.4573740D 01	0.2951474D 00	0.9999537D 00	0.4629481D-04	0.3076956D-04
6	0.4459693D 01	0.2911462D 00	0.9999022D 00	0.9778794D-04	0.5149312D-04
10	0.4356212D 01	0.2875790D 00	0.9998108D 00	0.1891729D-03	0.9138456D-04
12	0.4314488D 01	0.2861642D 00	0.9997544D 00	0.2455793D-03	0.5640645D-04
15	0.4261991D 01	0.2844098D 00	0.9996604D 00	0.3396045D-03	0.9402520D-04

TABLE (4.5) -- CONTINUED

M A R G I N A L P R O B A B I L I T I E S
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TIME IN YEARS -----	STATE 1 -----	STATE 2 -----	STATE 3 -----	STATE 4 -----	STATE 5 -----	STATE 6 -----	STATE 7 -----	STATE 8 -----	STATE 9 -----	STATE 10 -----	SUM ---
1	0.4169221	0.2150030	0.1796042	0.1120639	0.0522217	0.0181717	0.0047208	0.0009154	0.0001325	0.0000155	0.9997708
3	0.3055484	0.2118150	0.2068756	0.1500964	0.0808877	0.0323736	0.0096207	0.0021224	0.0003475	0.0000463	0.9997378
6	0.2347309	0.1987530	0.2176085	0.1762804	0.1056518	0.0468428	0.0153608	0.0037247	0.0006677	0.0000978	0.9997184
10	0.1785750	0.1802846	0.2195185	0.1970469	0.1303895	0.0635986	0.0228615	0.0060549	0.0011813	0.0001892	0.9997040
12	0.1583981	0.1714325	0.2180335	0.2041268	0.1406753	0.0713571	0.0266367	0.0073156	0.0014778	0.0002456	0.9996590
15	0.1351041	0.1594367	0.2143236	0.2116882	0.1536277	0.0819129	0.0320830	0.0092286	0.0019490	0.0003396	0.9996535
UPPER BOUND	5.030000	4.849900	4.549900	4.249900	3.949900	3.649900	3.349900	3.049900	2.749900	2.449900	
LOWER BOUND	4.850000	4.550000	4.250000	3.950000	3.650000	3.350000	3.050000	2.750000	2.450000	0.0	

THE EXPECTED VALUE OF LIFE IS 0.34178980-02 YEARS

TABLE (4.5) -- CONTINUED

INPUT DATA FOR RUN 6

A. SYSTEM GEOMETRY

THE DEPTH OF THE SURFACE LAYER IS 3.0000 INCHES
THE DEPTH OF THE BASE LAYER IS 3.0000 INCHES

B. MECHANICAL PROPERTIES

NORMAL DEFLECTION SERIES (INCHES)
AT THE SURFACE BELOW THE LOAD CENTER

RADIAL STRAIN SERIES (INCHES/INCH)
AT THE FIRST INTERFACE UNDER THE LOAD CENTER

I	COEFFICIENTS (G(I))	EXPONENTS (DELTA(I) 1./SEC)	COEFFICIENTS (GRAD(I))	EXPONENTS (DELTA(I) 1./SEC)
1	0.1575696C-02	0.50000000 03	0.36773290-04	0.50000000 03
2	-0.10529300-01	0.50000000 02	-0.27613620-03	0.50000000 02
3	-0.11256250 00	0.50000000 01	-0.34657570-02	0.50000000 01
4	-0.73709190-01	0.50000000 00	-0.33991920-02	0.50000000 00
5	-0.8947927C-01	0.50000000-01	-0.29370020-02	0.50000000-01
6	-0.84190850-01	0.50000000-02	-0.27335620-02	0.50000000-02
7	-0.3589208C-01	0.50000000-03	-0.15258070-02	0.50000000-03
8	-0.87608400-01	0.50000000-04	-0.24755710-02	0.50000000-04
9	-0.8049774C-01	0.50000000-05	-0.20244790-02	0.50000000-05
10	-0.95255850-01	0.50000000-06	-0.24268630-02	0.50000000-06
11	-0.65539360-01	0.50000000-07	-0.17868280-02	0.50000000-07
12	-0.13313290 00	0.50000000-08	-0.35328870-02	0.50000000-08
13	0.39535520-01	0.50000000-09	0.10204320-02	0.50000000-09
14	-0.38974000 00	0.50000000-10	-0.10282520-01	0.50000000-10
15	0.15771420 01	0.0	0.41828560-01	0.0

THE SUM OF THE COEFFICIENTS IS 0.31011580 00

THE SUM OF THE COEFFICIENTS IS 0.56191210-02

THE SPATIAL COEFFICIENT OF VARIATION IS 0.50000000 00

THE SPATIAL COEFFICIENT OF VARIATION IS 0.50000000 00

SPATIAL CORRELATION PARAMETERS: B = 1.0000 C = 0.5000

TABLE (4.6): ANALYSIS OF SYSTEM 1; WITH THIN GEOMETRY, MEDIUM PROPERTIES, AND POOR QUALITY CONTROL

FATIGUE PROPERTIES

TIME IN YEARS	TEMPERATURE IN DEGREES FAHRENHEIT	COEFFICIENT K1		EXPONENT K2	
		MEAN	VARIANCE	MEAN	VARIANCE
1	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
2	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
3	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
4	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
5	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
6	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
7	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
8	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
9	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
10	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
11	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
12	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
13	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
14	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01
15	74.3000	0.17579160-10	0.49444290-22	0.38280010 01	0.23445740 01

THE CORRELATION COEFFICIENT OF K1 AND K2 IS 0.0

C. LOAD CHARACTERISTICS

RADIUS OF THE LOAD = 6.4000 INCHES

MEAN LOAD INTENSITY = 80.0000 PSI

	MEAN	VARIANCE
LOAD AMPLITUDE (INTENSITY MULTIPLIES)	0.10000000 01	0.25000000 00
DURATION OF THE LOADS (SECONDS)	0.50000000-01	0.90000000-03

MEAN RATE OF (PCISSON) TRAFFIC LOADS: 22500. LCADS/MONTH

TABLE (4.6) -- CONTINUED

D. TEMPERATURES

TIME IN YEARS	TEMPERATURE IN DEGREES FEHRENHEIT	TEMPERATURE SHIFT FACTORS, GAMMAS (FOR THE TIME - TEMPERATURE SUPERPOSITION)
1	74.3000	0.57147880 00
2	74.3000	0.57147880 00
3	74.3000	0.57147880 00
4	74.3000	0.57147880 00
5	74.3000	0.57147880 00
6	74.3000	0.57147880 00
7	74.3000	0.57147880 00
8	74.3000	0.57147880 00
9	74.3000	0.57147880 00
10	74.3000	0.57147880 00
11	74.3000	0.57147880 00
12	74.3000	0.57147880 00
13	74.3000	0.57147880 00
14	74.3000	0.57147880 00
15	74.3000	0.57147880 00

REFERENCE TEMPERATURE = 77.0000

TABLE (4.6) -- CONTINUED

***** OUTPUT DATA FOR RUN 6 *****

R E S P O N S E H I S T O R Y

TIME IN YEARS	RUT DEPTH		SLOPE VARIANCE		STRAIN		CRACKS	
	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE
1	0.77492760-05	0.15026590-10	C.19217290-08	0.10836360-19	0.58085720-02	0.88819650-05	0.82787680 01	0.11197590 01
3	0.10000010-04	0.15725030-10	C.20077660-08	0.11322990-19	0.58085720-02	0.88819650-05	0.24836300 02	0.32592770 01
6	0.11499620-04	0.15958720-10	C.20326260-08	0.11463260-19	0.58085720-02	0.88819650-05	0.49672610 02	0.67185550 01
10	0.12766990-04	0.16115740-10	C.20459830-08	0.11538090-19	0.58085720-02	0.88819650-05	0.82787680 02	0.11197590 02
12	0.13244380-04	0.16172030-10	C.20498140-08	0.11559350-19	0.58085720-02	0.88819650-05	0.99345210 02	0.13437110 02
15	0.13818380-04	0.16241240-10	C.20536130-08	0.11580170-19	0.58085720-02	0.88819650-05	0.12418150 03	0.16796390 02

P E R F O R M A N C E C H A R A C T E R I S T I C S

TIME IN YEARS	SERVICEABILITY		RELIABILITY	CUMULATIVE DISTRIBUTION OF LIFE	PROBABILITY MASS FUNCTION OF LIFE
	MEAN	VARIANCE			
1	0.31786440 01	C.11387920 01	0.75266280 00	0.24733720 00	0.24733720 00
3	0.25775240 01	0.12437490 01	0.54555430 00	0.45444570 00	0.20710850 00
6	0.21035660 01	C.13301320 01	0.38197620 00	0.61802380 00	0.16357810 00
10	0.16539430 01	0.14144110 01	0.25166130 00	0.74833870 00	0.13031450 00
12	0.14724710 01	0.14490250 01	0.20840040 00	0.79159960 00	0.43260880-01
15	0.12448380 01	0.14928330 01	0.16199580 00	0.83800420 00	0.46404630-01

TABLE (4.6) ← CONTINUED

M A R G I N A L P R O B A B I L I T I E S

TIME IN IN YEARS	STATE 1	STATE 2	STATE 3	STATE 4	STATE 5	STATE 6	STATE 7	STATE 8	STATE 9	STATE 10	SUM
1	0.0586511	0.0407213	0.0583019	0.0771696	0.0544306	0.1068268	0.1117255	0.1080248	0.0965605	0.2473372	0.9997493
3	0.0207916	0.0176788	0.0283685	0.0423623	0.0588686	0.0761292	0.0916180	0.1026057	0.1069369	0.4544457	0.9598054
6	0.0086248	0.0083246	0.0144657	0.0233195	0.0352827	0.0499092	0.0660055	0.0816131	0.0943449	0.6180238	0.9598579
10	0.0036010	0.0038417	0.0070776	0.0122395	0.0198677	0.0302719	0.0432955	0.0581240	0.0732455	0.7483387	0.9999031
12	0.0025094	0.0027749	0.0052304	0.0092681	0.0154386	0.0241762	0.0355902	0.0492535	0.0640779	0.7915996	0.9999188
15	0.0015855	0.0018280	0.0035400	0.0064561	0.0110889	0.0179371	0.0273250	0.0392026	0.0529684	0.8380042	0.9999359
UPPER BOUND	5.030000	4.849900	4.549900	4.249900	3.949900	3.649900	3.349900	3.049900	2.749900	2.449900	
LOWER BOUND	4.850000	4.550000	4.250000	3.950000	3.650000	3.350000	3.050000	2.750000	2.450000	0.0	

THE EXPECTED VALUE OF LIFE IS 0.43684810 01 YEARS

TABLE (4.6) -- CONTINUED

INPUT DATA FOR RUN 7

A. SYSTEM GEOMETRY

THE DEPTH OF THE SURFACE LAYER IS 3.0000 INCHES
THE DEPTH OF THE BASE LAYER IS 3.0000 INCHES

B. MECHANICAL PROPERTIES

NORMAL DEFLECTION SERIES (INCHES)
AT THE SURFACE BELOW THE LOAD CENTER

RADIAL STRAIN SERIES (INCHES/INCH)
AT THE FIRST INTERFACE UNDER THE LOAD CENTER

I	COEFFICIENTS (G(I))	EXPONENTS (DELTA(I) 1./SEC)	COEFFICIENTS (GRAD(I))	EXPONENTS (DELTA(I) 1./SEC)
1	0.1575696D-02	0.5000000E 03	0.3677329D-04	0.5000000E 03
2	-0.1052930D-01	0.5000000D 02	-0.2761362D-03	0.5000000E 02
3	-0.1125625E 00	0.5000000E 01	-0.3465797D-02	0.5000000E 01
4	-0.7370919D-01	0.5000000D 00	-0.3399192D-02	0.5000000E 00
5	-0.8947927E-01	0.5000000E-01	-0.2937002D-02	0.5000000E-01
6	-0.9419085D-01	0.5000000D-02	-0.2733562D-02	0.5000000E-02
7	-0.8589208E-01	0.5000000D-03	-0.1925807D-02	0.5000000E-03
8	-0.8760840E-01	0.5000000D-04	-0.2475571D-02	0.5000000E-04
9	-0.8049774D-01	0.5000000D-05	-0.2024479D-02	0.5000000E-05
10	-0.9525585E-01	0.5000000D-06	-0.2426863D-02	0.5000000E-06
11	-0.6553536E-01	0.5000000E-07	-0.1786828D-02	0.5000000E-07
12	-0.1331329D 00	0.5000000D-08	-0.3532887D-02	0.5000000E-08
13	0.3953552E-01	0.5000000E-09	0.1020432D-02	0.5000000E-09
14	-0.3897400D 00	0.5000000D-10	-0.1028252D-01	0.5000000E-10
15	0.1577142E 01	0.0	0.4182856D-01	0.0

THE SUM OF THE COEFFICIENTS IS 0.3101158D 00

THE SUM OF THE COEFFICIENTS IS 0.5619121D-02

THE SPATIAL COEFFICIENT OF VARIATION IS 0.1000000D 00

THE SPATIAL COEFFICIENT OF VARIATION IS 0.1000000D 00

SPATIAL CORRELATION PARAMETERS: B = 1.0000 C = 0.5000

TABLE (4.7): ANALYSIS OF SYSTEM 1; WITH THIN GEOMETRY, MEDIUM PROPERTIES, AND GOOD QUALITY CONTROL

FATIGUE PROPERTIES

TIME IN YEARS	TEMPERATURE IN DEGREES FAHRENHEIT	COEFFICIENT K1		EXPONENT K2	
		MEAN	VARIANCE	MEAN	VARIANCE
1	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
2	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
3	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
4	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
5	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
6	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
7	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
8	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
9	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
10	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
11	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
12	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
13	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
14	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00
15	74.3000	0.17579160-10	0.30902680-23	0.38280010 01	0.14653590 00

THE CORRELATION COEFFICIENT OF K1 AND K2 IS 0.0

C. LCAC CHARACTERISTICS

RADIUS OF THE LOAD = 6.4000 INCHES

MEAN LOAD INTENSITY = 80.0000 PSI

	MEAN	VARIANCE
LOAD AMPLITUDE (INTENSITY MULTIPLIES)	0.10000000 01	0.10000000-01
DURATION OF THE LOADS (SECONDS)	0.50000000-01	0.10000000-03

MEAN RATE OF (POISSON) TRAFFIC LOADS: 22500. LOADS/MONTH

TABLE (4.7) -- CONTINUED

D. TEMPERATURES

TIME IN YEARS	TEMPERATURE IN DEGREES FAHRENHEIT	TEMPERATURE SHIFT FACTORS, GAMMAS (FOR THE TIME - TEMPERATURE SUPERPOSITION)
1	74.3000	0.57147880 00
2	74.3000	0.57147880 00
3	74.3000	0.57147880 00
4	74.3000	0.57147880 00
5	74.3000	0.57147880 00
6	74.3000	0.57147880 00
7	74.3000	0.57147880 00
8	74.3000	0.57147880 00
9	74.3000	0.57147880 00
10	74.3000	0.57147880 00
11	74.3000	0.57147880 00
12	74.3000	0.57147880 00
13	74.3000	0.57147880 00
14	74.3000	0.57147880 00
15	74.3000	0.57147880 00

REFERENCE TEMPERATURE = 77.0000

TABLE (4.7) -- CONTINUED

***** OUTPUT DATA FOR RUN 7 *****

RESPONSE HISTORY

TIME IN YEARS	RUT DEPTH		SLOPE VARIANCE		STRAIN		CRACKS	
	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE	EXPECTED VALUE	VARIANCE
1	0.7749276D-05	0.6090193D-12	C.7686834D-10	0.8286586D-20	0.5801302D-02	0.3559115D-06	0.1401526D 03	0.2854663D 04
3	0.1000001D-04	0.6529375D-12	C.8030816D-10	0.8658621D-20	0.5801302D-02	0.3559115D-06	0.4204577D 03	0.8563988D 04
6	0.1149562D-04	0.6862977D-12	C.8130004D-10	0.8765749D-20	0.5801302D-02	0.3559115D-06	0.8409155D 03	0.1712798D 05
10	0.1276699D-04	0.7246229D-12	C.8183097D-10	0.8822792D-20	0.5801302D-02	0.3559115D-06	0.1401526D 04	0.2854663D 05
12	0.1324438D-04	0.7429025D-12	C.8198256D-10	0.8838958D-20	0.5801302D-02	0.3559115D-06	0.1681831D 04	0.3425595D 05
15	0.1381838D-04	0.7697180D-12	C.8213201D-10	0.8854749D-20	0.5801302D-02	0.3559115D-06	0.2102289D 04	0.4281954D 05

PERFORMANCE CHARACTERISTICS

TIME IN YEARS	SERVICEABILITY		RELIABILITY	CUMULATIVE DISTRIBUTION OF LIFE	PROBABILITY MASS FUNCTION OF LIFE
	MEAN	VARIANCE			
1	0.4003557D 01	0.5667828D-01	0.1000000D 01	0.3394464D-10	0.3394464D-10
3	0.3367080D 01	0.6134700D-01	0.9998935D 00	0.1065478D-03	0.1065478D-03
6	0.2838014D 01	C.6578152D-01	0.9348902D 00	0.6510976D-01	0.6500321D-01
10	0.2329516D 01	0.7112926D-01	0.3258575D 00	0.6741425D 00	0.6090328D 00
12	0.2122469D 01	C.7371133D-01	0.1139057D 00	0.8860943D 00	0.2119518D 00
15	0.1859718D 01	0.7743341D-01	0.1696479D-01	0.9830352D 00	0.9694086D-01

TABLE (4.7) -- CONTINUED

M A R G I N A L P R O B A B I L I T I E S

TIME IN YEARS	STATE 1	STATE 2	STATE 3	STATE 4	STATE 5	STATE 6	STATE 7	STATE 8	STATE 9	STATE 10	SUM
1	0.0001887	0.0106693	0.1394262	0.4386006	0.3420806	0.0656815	0.0029888	0.0000309	0.0000001	0.0000000	0.9996667
3	0.0000000	0.0000009	0.0001813	0.0091166	0.1173627	0.4007332	0.3721099	0.0938076	0.0062480	0.0001065	0.9996667
6	0.0000000	0.0000000	0.0000000	0.0000073	0.0007658	0.0221813	0.1812764	0.4298978	0.3004327	0.0651098	0.9996710
10	0.0	0.0000000	0.0000000	0.0000000	0.0000004	0.0000647	0.0033866	0.0539867	0.2682368	0.6741425	0.9998177
12	0.0	0.0	0.0000000	0.0000000	0.0000000	0.0000031	0.0003143	0.0100885	0.1034181	0.8860943	0.9999184
15	0.0	0.0	0.0	0.0000000	0.0000000	0.0000000	0.0000094	0.0006793	0.0162601	0.9830352	0.9999840
UPPER BOUND	5.030000	4.849900	4.549900	4.249900	3.949900	3.649900	3.349900	3.049900	2.749900	2.449900	
LOWER BOUND	4.850000	4.550000	4.250000	3.950000	3.650000	3.350000	3.050000	2.750000	2.450000	0.0	

THE EXPECTED VALUE OF LIFE IS 0.10478200 02 YEARS

TABLE (4.7) -- CONTINUED